INTERNATIONAL PROGRAM (MASTER)

CLASSES: SUMMER SEMESTER 2009

1520 Algebra II
Lecture: 4 h, 8 credit points; Tue 09:45-11:15 S 34, Fr 09:45-11:15 S 11
Tutorial: 2 h, 2 credit points; Tue 15:45-17:15 S 12
PD Stefan Kühnlein

1562 Mathematical Theory of Maxwell's Equations
Lecture: 4 h, 8 credit points; Mo 09:45-11:15 S 31, Wed 09:45-11:15 S 31
Prof. Andreas Kirsch

1563 Sobolev Spaces
Lecture: 2 h, 4 credit points; Do 08:00-09:30 S 31
Prof. Wolfgang Reichel

1575 Boundary and Eigenvalue Problems
Lecture: 4 h, 8 credit points; Tue 11:30-13:00 S 31, Thu 11:30-13:00 S 31
Tutorial: 2 h, 2 credit points; Wed 15:45-17:15 S 31
Prof. Michael Plum

1596 Applied Stochastic Models
Lecture: 4 h, 8 credit points; Mo 14:00-15:30 S 34, Wed 14:00-15:30 S 33
Tutorial: 2 h, 2 credit points; Tue 15:45-17:15 S 31
Prof. P.R. Parthasarathy

1608 Geometric numerical integration
Lecture: 2 h, 4 credit points; Fr 11:30-13:00 S 34
Prof. Tobias Jahnke

Time-table

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30				Sobolev	
09:45-11:15	Maxwell	Algebra II	Maxwell		Algebra II
11:30-13:00	ConvGeo	BEP	Queues	BEP	GNE
14:00-15:30	AppStoch		AppStoch		
15:45-17:15		Algebra II (T) AppStoch (T)	BEP (T)		

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: SUMMER SEMESTER 2009 CLASSES

1520 Algebra II

Lecture: 4 h, 8 credit points; Tue 09:45-11:15 S 34, Fr 09:45-11:15 S 11
Tutorial: 2 h, 2 credit points; Tue 15:45-17:15 S 12
PD Stefan Kühnlein

Contents: We will spend the first few weeks by learning the basic language of category theory. This gives a general framework for those structural properties of (homo-)morphisms which do not (or not necessarily) have to do with elements. We will discuss functors and representability and in particular will cover the Yoneda-Lemma. We will then be able to define the concept of a universal mapping property in a very precise way.

In the remaining time we will discuss several aspects of module theory. Important entries in our vocabulary will be tensor products, noetherian modules, Hilbert's Basissatz, reducibility, orders and integral ring extension. The final section will be devoted to the basics of representation theory of infnite groups, i.e. given a group G and a field K one wants to unterstand all homomorphisms from G to $\mathrm{GL}_n\ (K)$ (with varying n).

Textbooks:
There will be a script by my own (german)
Jacobson, *Basic Algebra II*, Freeman 1980
Lang, *Algebra*, Addison Weseley 1984
Jantzen, Schwermer, *Algebra*, Springer 2006

1562 Mathematical Theory of Maxwell's Equations

Lecture: 4 h, 8 credit points; Mo 09:45-11:15 S 31, Wed 09:45-11:15 S 31
Prof. Andreas Kirsch

Contents:

- **1: Introduction:** Derivation of the equations, constitutive equations, special cases: electrostatics, magnetostatics, time-harmonic fields, boundary and transmission conditions
- **2: Integral Equation Methods for Time-Harmonic Scattering Problems:** Representation theorems, jump conditions of potentials, functional analytic tools, boundary integral equation, volume integral equation
- **3: Sobolev Spaces:** Sobolev spaces of scalar functions, the Sobolev space $H(\mathrm{curl}, D)$, imbedding and trace theorems
- **4: The Cavity Problem:** Variational formulation, regularity results, functional analytic tools, existence and uniqueness

Textbooks:
D. Colton and R. Kress, *Integral Equation Methods in Scattering Theory*, Wiley 1983
D. Colton and R. Kress, *Inverse Acoustic and Electromagnetic Scattering Theory, 2nd ed.*, Springer 1998
P. Monk, *Finite Element Methods for Maxwell's Equations*, Oxford University Press 2003
J. C. Nédélec, *Acoustic and Electromagnetic Equations*, Springer 2001

1563 Sobolev Spaces

Lecture: 2 h, 4 credit points; Do 08:00-09:30 S 31
Prof. Wolfgang Reichel

Contents: Sobolev spaces are Banach spaces of functions, which possess derivatives in the so called *weak sense*. Theses function spaces play an important role in the modern theory of partial differential equations (PDEs). Many properties of theses functions are frequently used – but less frequently proved in lectures on PDEs.

In the lecture the following topics (not necessarily in the same order) will be discussed and proofs of the main theorems will be provided:

- Definition of weak derivatives, definition of Sobolev spaces
- Embedding theorems (continuous and compact embeddings)
- Poincaré- and Hardy-inequalities
- Applications to PDEs
- Trace theorems, extension theorems and applications
- H_{div}, H_{curl}-spaces and applications

Literature:
Adams and Fournier, *Sobolev spaces, 2nd ed.*, Elsevier 2003
Evans and Gariepy, *Measure theory and fine properties of functions*, CRC Press 1992

1575 Boundary and Eigenvalue Problems

1575 Boundary and Eigenvalue Problems

Lecture: 4 h, 8 credit points; Tue 11:30-13:00 S 31, Thu 11:30-13:00 S 31
Tutorial: 2 h, 2 credit points; Wed 15:45-17:15 S 31
Prof. Michael Plum

Contents: A boundary value problem consists of an elliptic (or ordinary) differential equation posed on some domain, together with additional conditions required on the boundary of the domain, e.g. prescribed values for the unknown function. In the case of an ordinary differential equation, these "boundary conditions" are posed at both ends of the underlying interval (in contrast to initial value problems). Typical origins of boundary value problems are steady-state (i.e. time-independent) situations in physics and engineering.

An eigenvalue problem for a differential equation is a linear and homogeneous boundary value problem depending (typically linearly) on an additional parameter, and one is interested in values of this parameter such that the boundary value problem has nontrivial solutions. Eigenvalue problems arise e.g. after separation of variables in time-dependent problems (thus describing many vibrational situations, including quantum mechanics) or in stability and critical value theory for mathematical and physical systems.

The lecture course will start with a series of examples for occurrence of boundary value problems in mathematical physics, followed by the (comparatively simple) existence theory for ordinary linear regular boundary value problems. A large part of the lecture course will then be covered by an existence theory for linear elliptic boundary value problems; for this purpose, weak formulations of boundary value problems, Sobolev spaces, trace theory, the Lax-Milgram Lemma, Garding's inequality, Fredholm's Alternative, and other tools will be introduced. In a natural way, this theory connects to eigenvalue problems. Based upon the Spectral Theorem for compact symmetric operators in Hilbert spaces, and on the existence theory for linear boundary value problems, an eigenvalue theory for symmetric elliptic differential operators will be presented. If time permits, the lecture course closes with some numerical methods for boundary and eigenvalue problems (Galerkin, Finite Elements).

The lecture course addresses students in their fourth semester (second year) or higher, with substantial knowledge in analysis and linear algebra. The course is suitable for students of mathematics, and for students of other subjects who have strong mathematical interests.

The lectures will be accompanied by exercise lessons. Attendance of these exercises is strongly recommended to all participants.

Textbooks:

A. Friedman, *Partial Differential Equations* (general elliptic PDE of order 2m, but smooth data only)

D. Gilbarg, N. Trudinger, *Elliptic Partial Differential Equations of Second Order* (elliptic PDE of second order, mainly Dirichlet b.c.)

R. A. Adams, *Sobolev Spaces* (no PDEs, but excellent and general introduction into Sobolev spaces, an essential tool in PDE theory)

L. C. Evans, *Differential Partial Equations*

1596 Applied Stochastic Models

Lecture: 4 h, 8 credit points; Mo 14:00-15:30 S 34, Wed 14:00-15:30 S 33
Tutorial: 2 h, 2 credit points; Tue 15:45-17:15 S 31
Prof. P.R. Parthasarathy

Contents: Reliability Theory: Replacements and maintenance; discounting; Group replacements; Series and parallel systems; System reliability and availability, Simple reliability models

Simulation: Random numbers and Monte Carlo simulation; Generating discrete and continuous random variables; Simulation with process models, queueing models, Statistical analysis of simulated data; Variance reduction; Goodness of fit; Markov chain Monte Carlo.

Quality Control: Acceptance Sampling for attributes and for variables; Operating characteristics and average run length; Single, double and sequential plans; Control charts, their construction and use.

Textbooks:
H.A. Taha, *Operations Research, an Introduction, 7th ed.*, Prentice Hall, New Jersey 2003
S. M. Ross, *Simulation, 3rd ed.*, Academic Press, San Diego 2003
S. M. Ross, *Probability Models, 9th ed.*, Elsevier, Bulrlington 2007
R.A. Johnson, *Miller and Freund's Probability and Statistics for Engineers, 7th ed.*, Prentice-Hall, New Jersey

1608 Geometric numerical integration

Lecture: 2 h, 4 credit points; Fr 11:30-13:00 S 34
Prof. Tobias Jahnke

Contents: Many ordinary differential equations have certain structural properties. The exact flow of the differential equation can, for example, be reversible, symplectic, or volume-conserving, and quantities such as the energy, the angular momentum, or the norm of the solution can remain constant although the solution itself changes in time. It is desirable to preserve these geometric properties when the solution or the flow is approximated by a numerical integrator, because reproducing the correct qualitative behaviour is important in most applications. It turns out, however, that many numerical methods destroy the structure of the solution, and only selected methods allow to preserve the geometric properties of the exact flow. These methods are called geometric numerical integrators.

In this lecture we will investigate

- why certain methods are (or are not) geometric numerical integrators,
- how to construct geometric numerical integrators,
- which properties are conserved, and in which sense,
- how structure conservation is related to the long-time error behaviour of the method.

We will mainly focus on geometric integrators for Hamiltonian systems. The lecture will be suited for students in mathematics, physics, and other sciences with a basic knowledge of ordinary differential equations.

Textbooks:
E. Hairer, Ch. Lubich, G. Wanner, *Geometric numerical integration. Structure-preserving algorithms for ordinary differential equations. 2nd ed*, Springer 2006
E. Hairer, Ch. Lubich, G. Wanner, *Geometric numerical integration illustrated by the Störmer-Verlet method. Acta Numerica 12, 399-450 (2003).*