

**MASTER DEGREE COURSES OFFERED IN ENGLISH IN
WINTER SEMESTER 2021-22**

• **0104500 Graph Theory**

Instructor: Maria Axenovich

Weekly Hours: 4+2

Graphs are basic structures in discrete mathematics that in particular model various networks. The course starts with basic concepts in graph theory such as trees, cycles, matchings, factors, connectivity, and their interconnections. Further topics include properties of graphs with forbidden subgraphs, planar graphs, graph colorings, random graphs, Ramsey theory, and graph minors. Not only classical, but very recent results in the field will be discussed. The class is oriented towards problem solving. Particular attention to proof writing techniques will be paid in the problem classes. The final grade will be based on the written exam. Bonus points will be given for weekly or biweekly homework assignments.

Prerequisites: Basic knowledge of linear algebra

Lecture: Mon 10:00-11:30, Wed 12:00-13:30,

Problem class: Fri 14:00-15:30

• **0105300 Classical Methods for PDEs**

Instructor: Tobias Lamm

Weekly Hours: 4+2

In this introductory course to the theory of Partial Differential Equations we start by studying the basic model equations in great detail. These include harmonic functions, the heat equation and the wave equations. Afterwards we introduce and study Sobolev spaces and develop the L^2 -regularity theory for elliptic equations.

Prerequisites: Calculus 1-3

Lecture: Tu 10:00-11:30, Th 10:00-11:30

Problem class: Wed 14:00-15:30

- **0106000 Lie algebras**

Instructor: Tobias Hartnick

Weekly Hours: 4+2

Lie theory, i.e. the study of Lie groups and Lie algebras is a beautiful and central topic in modern mathematics and theoretical physics. It had its origins in the nineteenth century in Lies idea of applying Galois theory to differential equations and in Kleins Erlanger Programm of treating symmetry groups as the fundamental objects in geometry. This is the first part of a two-semester introductory course to Lie theory which focuses purely on the algebraic parts of the theory, i.e. the structure and representation theory of Lie algebras, and therefore can be attended with only minimal prerequisites. Although the course it self-contained, it will be continued in the summer term with a course on the (more demanding) geometric and analytic parts of Lie theory (also in English).

Prerequisites: A secure grasp of linear algebra is essential. Further experience with algebraic structures (e.g. basic group theory or geometry) is useful, but not strictly necessary. Although connections to physics will be hinted at during the course, no prior knowledge of physics is required.

Lecture: Mo 14:00-15:30, Th 12:00-13:30

Problem class: Tu 8:00-9:30

- **0106200 Fourier Analysis and its applications to PDEs**

Instructor: Xian Liao

Weekly Hours: 3+1

In this course we will introduce the theory of Fourier analysis and then show its applications to some typical PDEs. The Fourier transform and the Littlewood-Paley decomposition techniques have shown their efficiency in the study of evolutionary equations. As examples we will study the transport-diffusion equations, Navier-Stokes equations and wave-type equations.

Prerequisites: Basic concepts from functional analysis.

Lecture / Problem classes: Wed 10:00-11:30, Fr 12:00-13:30.

• **0106700 Introduction to microlocal analysis**

Instructor: Ruoci Sun

Weekly Hours: 2

This course is concerned with the theory of microlocal analysis. I will introduce the main notions about the pseudo-differential operators, symbolic calculus, the wave front set, and the microlocal defect measures and describe some of the main results, such as continuity, Gårding's inequalities, and propagation of singularities.

Prerequisites: Functional analysis, in particular the distribution theory, Fourier transform in the Schwartz class and tempered distributions and Sobolev spaces.

Lecture: Thursday 8:00-9:30.

References:

- *Opérateurs pseudo différentiel et théorème de NashMoser*, by Serge Alinhac and Patrick Gérard.
- *Tools and Problems in Partial Differential Equations*, by Thomas Alazard and Claude Zuily.
- *The Analysis of Linear Partial Differential Operators*, Vol. 3, by Lars Hörmander.

• **0107000 Numerical Analysis of Helmholtz Problems**

Instructor: Barbara Verfürth

Weekly Hours: 2

The lecture deals with the (numerical) analysis for Helmholtz problems on bounded domains using finite element methods. The Helmholtz equation is an important model in time-harmonic wave propagation. Its analytical and numerical treatment is challenging since the equation is indefinite and typical solutions show oscillations. Contents include modeling of Helmholtz

problems, well-posedness analysis of the variational formulation and a priori error estimates for finite element discretizations.

Prerequisites: Basic knowledge about partial differential equations and numerical methods for them. Knowledge about finite element methods is recommended, but not compulsory. (The lecture Finite Element Method can be attended in parallel).

Lecture: Th 16.00-17.30

- **0108600 Exponential Integrators**

Instructors: Benjamin Dörich and Jan Leibold

Weekly Hours: 3+1

In this class we consider the construction, analysis, implementation and application of exponential integrators. The focus will be on two types of stiff problems. The first one is characterized by a Jacobian that possesses eigenvalues with large negative real parts. Parabolic partial differential equations and their spatial discretization are typical examples. The second class consists of highly oscillatory problems with purely imaginary eigenvalues of large modulus. Apart from motivating the construction of exponential integrators for various classes of problems, our main intention in this class is to present the mathematics behind these methods. We will derive error bounds that are independent of stiffness or highest frequencies in the system.

Prerequisites: Basics knowledge of finite element methods and numerical methods for differential equations. Some knowledge on functional analysis is also helpful.

Lecture: Mo 08:00-09:30, Fr 10:00-11:30 (biweekly)

Problem class: Fr 10:00-11:30 (biweekly)

- **0109400 Mathematical Modelling and Simulation**

Instructor: Gudrun Thäter

Weekly Hours: 2+1

The general aim of this lecture course is threefold: 1) to interconnect different mathematical fields, 2) to connect mathematics and real life problems, 3) to learn to be critical and to ask relevant questions. During the lecture course there will be one lecture of a person from industry and one excursion (at the end of the lecture course). To earn the credits you have to attend the lecture, finish the work on one project during the term in a group of 2-3 persons and pass the exam. The topic of the project is up to the choice of each group. The exam is an oral exam. Please register with the ILIAS-course in order to be kept up to date with all material and information - especially since due to the changing Covid-situation it is not clear if and when I can teach in person.

There are no special prerequisites.

Lecture: Mon 10:00-11:30

Problem class: Tu 16:00-17:30

Literature: Hans-Joachim Bungartz e.a.: Modeling and Simulation: An Application-Oriented Introduction, Springer, 2013 (English)

- **0110300 Finite Element Methods**

Instructor: Willy Dörfler

Weekly hours: 4+2

The topic of the course is to provide knowledge in numerically solving elliptic boundary value problems using the finite element method. We first present some basics like the weak formulation of these boundary value problems. Then we study the finite element approach, in its various aspects, such as grid generation, choice of ansatz spaces, datastructures for implementation, numerical integration, error estimates, a posteriori error estimates and solution techniques for the resulting systems of equations. A finite element solver in Matlab is provided to also get some practical experience.

Prerequisites: Numerics 1-2, Numerics of Differential equations or Introduction to Scientific Computing, Introductory course in Analysis of PDEs.

Lecture/ Problem class: Mon 12:00-13:30, Tu 14-15:30, Wed 16-17:30

Literature:

Dörfler, W: Accompanying Lecture Notes. Braess, D: Finite Elements: Theory, fast solvers, and applications in solid mechanics Brenner, SC and Scott, LR: The mathematical theory of finite element methods. Ciarlet, PG: The finite element method for elliptic problems.

- **0118000 Asymptotic Stochastics**

Instructor: Vicky Fasen-Hartmann

Weekly Hours: 4+2

In this class we extend the classical central limit theorem of Lindeberg-Feller to the multivariate setting and get rid of the iid assumption. We use these results to prove the consistency and asymptotic normality of both parametric estimators (e.g. maximum-likelihood estimator) and nonparametric estimators (e.g. U-statistics), to construct asymptotic confidence intervals and consistent tests (e.g. likelihood ratio test). Moreover, we derive limit results in metric spaces and apply them as well in statistics.

Prerequisites: Einführung in die Stochastik (Introduction to Probability and Statistics), Wahrscheinlichkeitstheorie (Probability Theory)

Lecture: Tu 12:00-13:30, Wed 8:00-9:30

Problem class: Th 14:00-15:30

- **0163500 Mathematical Methods in Quantum Mechanics I**

Instructor: Dirk Hundertmark

Weekly hours: 4+2

This course provides an introduction to the mathematical theory of (non-relativistic) quantum mechanics. This is a beautiful theory, deep in the intersection of mathematics and physics. Not only did quantum mechanics influence the development of a large part of modern mathematics, but also conversely, modern mathematics can help a lot for understanding quantum mechanics: A bit of mathematical rigor can go a long way to untangle the usual hand waving arguments in theoretical physics.

We will cover one and many-particle systems in quantum mechanics, including Coulomb systems. When time permits, we will also study (some) aspects of scattering theory and/or Fock spaces and second quantisation.

The aim is to not only cover the basic mathematical foundations, such as self-adjointness, but also develop many more tools to rigorously study the physics of interacting many-particle systems. The course should be attractive to math students, who want to learn about quantum mechanics, and to physics students, who want to get a thorough introduction to the mathematical foundations of quantum mechanics.

Prerequisites: Basic concepts of analysis, functional analysis, and spectral theory are helpful, as is having some background in theoretical physics. However, as quoted in the introduction to the chapter “Preliminaries” in the book “Modern Methods in Mathematical Physics. Part I. Functional Analysis” by Reed and Simon: “The readers should not be discouraged if he finds that he does not have the prerequisites for reading the prerequisites” (P. Halmos). So parts of a script in spectral theory/functional analysis, which are relevant for this course, will be provided at the beginning of the course. More important than having mastered all the necessary prerequisites is the willingness of the students to learn and acquire them, when needed.

Lecture: Mo 16:00-17:30, Tu 12:00-13:30

Problem class: Fr 08:00-09:30

- **Seminar: Extremal Set Theory**
Mon 14:00-15:30
Instructors: Riasanovsky/Axenovich
- **Seminar: Completely integrable systems II**
Fr 14:00-15:30
Instructor: Liao
- **Seminar: Statistical Forecasting and Classification**
Tu 14:00-15:30
Instructor: Gneiting

There are more seminars available, please check with the instructors for a language choice.

For a current listing of courses and seminars in German and English, please see <https://www.math.kit.edu/vvz/seite/vvzkommend/de>