MASTER DEGREE COURSES OFFERED IN ENGLISH IN WINTER SEMESTER 2022-23

Courses are ordered by their catalog numbers:

- **0100027 Stochastic Simulation**
  Instructor: Sebastian Krumscheid
  Weekly Hours: 2+2

  The course covers mathematical concepts and computational tools used to analyze systems with uncertainty arising across various application domains. First, we will address stochastic modelling strategies to represent uncertainty in such systems. Then we will discuss sampling-based methods to assess uncertain system outputs via stochastic simulation techniques. The focus of this course will be on the theoretical foundations of the discussed techniques, as well as their methodological realization as efficient computational tools. Topics covered include the simulation of stochastic processes and Gaussian random fields, as well as Monte Carlo methods with a focus on variance reduction techniques and rare event simulations.

  Prerequisites: basic knowledge of Probability and Statistics, as well as of Numerical Analysis

  Lecture: Mo 11:30-13:00
  Problem class: Fr 14:00-15:30

- **0102948 Algebraic Topology**
  Instructor: Manuel Krannich
  Weekly Hours: 4+2

  This course provides an introduction to algebraic topology which is a fundamental discipline with ties to many areas in mathematics. Roughly, algebraic topology can be described as the study of topological spaces by means of algebraic invariants. An exemplary application is Brouwer's fixed point theorem: Every continuous self-map from the closed unit disc in d-dimensional Euclidean space has a fixed point. The proof relies on an algebraic invariant of topological spaces called singular homology which will be the main focus of this course. Further topics include basics of category theory and homological algebra, CW complexes, cellular homology, and Poincaré duality.

This course will be followed by a seminar in the following semester, which will be offered in English as well.

Prerequisites: Basic concepts of point-set topology and linear algebra.

Lecture: Mo 15:45-17:15, Wed 11:30-13:00
Problem class: Mo 14:00-15:30

- **0102956 Forecasting: Theory and Practice (Part I)**
  Instructor: Tilmann Gneiting
  Weekly Hours: 2+1

  A common desire of all humankind is to make predictions for the future. As the future is inherently uncertain, forecasts ought to be probabilistic, i.e., they ought to take the form of probability distributions over future quantities or events. In this class, which constitutes Part I of a two-semester series, we will study the probabilistic and statistical foundations of the science of forecasting.

  The goal in probabilistic forecasting is to maximize the sharpness of the predictive distributions subject to calibration, based on the information set at hand. Proper scoring rules such as the logarithmic score and the continuous ranked probability score serve to assess calibration and sharpness simultaneously, and relate to information theory and convex analysis. As a special case, consistent scoring functions provide decision-theoretically coherent tools for evaluating point forecasts. Throughout, concepts and methodologies will be illustrated in data examples and case studies.

  Prerequisites: A firm understanding of the contents of module Probability Theory is essential.

  Lecture: Tu 14:00-15:30
  Problem class: Th 8:00-9:30

- **0104800 Functional Analysis**
  Instructor: Xian Liao

Weekly hours: 4+2

Functional analysis is the study of normed complete vector spaces (called Banach spaces) and linear operators between them. It is built on the structure of linear algebra and analysis. Functional analysis finds its vast applications to e.g. partial differential equations, numerical analysis, probability. Contents of this lecture include definition and basic properties of Hilbert spaces, Lebesgue spaces, Sobolev spaces and linear functionals on Banach spaces.

Prerequisites: Linear algebra and analysis.

Lecture: Mo, Fr 11:30-13:00
Problem class: Wed 08:00-09:30

- **0105250 Random Graphs and Networks**
  Instructor: Daniel Hug
  Weekly Hours: 4+2

In the course, models of random graphs and networks are presented and methods will be developed which allow to state and prove results about the structure of such models. In particular, the following models are treated:
- Erdős–Renyi graphs
- Configuration models
- Preferential-Attachment graphs
- Generalized inhomogeneous random graphs
- Geometric random graphs

and the following methods are addressed:
- Branching processes
- Coupling arguments
- Probabilistic bounds
- Martingales
- Local convergence of random graph

Prerequisites: The contents of the module 'Probability Theory' are strongly recommended

Lecture: Tu 11:30-13:00, Th 15:45-17:15

Problem Class: Tu 08:00 - 09:30

- **0105500 Introduction to Stochastic Differential Equations**
  Instructor: Josef Janák
  Weekly Hours: 2+1

The students will learn fundamental results from the theory of stochastic differential equations in finite dimension. They will recognize fundamental examples for linear and non-linear stochastic differential equations and they will be able to apply basic solution concepts for such equations. They will also understand essential theorems of stochastic calculus and will be able to apply these to stochastic differential equations.

Students should be acquainted with the basics of continuous time stochastic processes, especially the Wiener process. Some basic knowledge on continuous-time martingales, the stochastic integral and Itô's formula would be favourable.

Prerequisites: Probability Theory (Wahrsheinlichkeitstheorie), Brownian Motion (Brownsche Bewegung), Continuous Time Finance (Finanzmathematik in stetiger Zeit)

Lecture: Fr 11:30-13:00
Problem class: Th 14:00-15:30

- **0105964 Introduction to Convex Integration**
  Instructor: Christian Zillinger
  Weekly Hours: 2+0

This lecture provides an introduction to the methods of convex integration and their applications:
- for isometric immersions,
- for the m-well problem in elasticity,
- for equations of fluid dynamics, and
- higher regularity of convex integration solutions.

Prerequisites: Basic concepts of partial differential equations and functional analysis

Lecture: Wed 11:30-13:00

Problem class: None

- **0108000 Bifurcation Theory**
  Instructor: Rainer Mandel
  Weekly hours: 4+2

  Bifurcation theory allows to find sufficient conditions for the existence of solution curves of parameter-dependent equations in Banach spaces. We are going to discuss applications to Ordinary and Partial Differential Equations. The topics that will be covered are the Implicit Function Theorem in Banach spaces, bifurcation from a simple eigenvalue (Crandall-Rabinowitz Theorem), bifurcation from infinity, and Hopf-bifurcation for periodic solutions of ordinary differential equations.

  Prerequisites: a thorough knowledge of analysis, ordinary differential equations, and the basics of functional analysis or boundary and eigenvalue problems.

  Lecture: Mo 09:45-11:15, Fr 08:00-09:30
  Problem class: Mo 15:45-17:15

- **0108100 Traveling waves**
  Instructor: Dr. Björn de Rijk
  Weekly Hours: 3+1

  Traveling waves are solutions to nonlinear partial differential equations (PDEs) that propagate over time with a fixed speed without changing their profiles. These special solutions arise in many applied problems where they model, for instance, water waves, nerve impulses in axons or light in optical fibers. Therefore, their existence and the naturally associated question of their dynamic stability is of interest, because only those waves which are stable can be observed in practice.

  The first step in the stability analysis is to linearize the underlying PDE about the wave and compute the associated spectrum, which is in general a nontrivial task. To approximate spectra associated with various waves, such as fronts, pulses and periodic wave trains, we introduce the following tools:
  - Sturm-Liouville theory
  - exponential dichotomies
  - Fredholm theory

- the Evans function
- parity arguments
- essential spectrum, point spectrum and absolute spectrum
- exponential weights.

The next step is to derive useful bounds on the linear solution operator, or semigroup, based on the spectral information. A complicating factor is that any non-constant traveling wave possesses spectrum up to the imaginary axis. For various dissipative PDEs, such as reaction-diffusion systems, we employ the bounds on the linear solution operator to close a nonlinear argument via iterative estimates on the Duhamel formula. For traveling waves in Hamiltonian PDEs, such as the NLS or KdV equation, we describe a different route towards stability based on the variational arguments of Grillakis, Shatah and Strauss.

Prerequisites: basic knowledge of (complex) analysis, linear algebra and ordinary differential equations, as well as some exposure to elementary functional analysis (i.e. normed and inner product spaces and the linear operators between them)

Lecture: Tu 08:00 - 09:30
Lecture/Problem class (will alternate): Wed 15:45 - 17:15

- **0109400 Mathematical Modelling and Simulation**
  Instructor: Gudrun Thäter
  Weekly Hours: 2+1

This course is open to mathematicians and engineers. The general aim of this lecture course is threefold:

1) to interconnect different mathematical fields,
2) to connect mathematics and real life problems,
3) to learn to be critical and to ask relevant questions.

During the lecture course there will be one lecture of a person from industry and one excursion (at the end of the lecture course) if possible.

To earn the credits you have to attend the lecture, finish the work on one project during the term in a group of 2-3 persons and pass the exam. The topic of the project is up to the choice of each group.

Prerequisites: Basic mathematical concepts from the first 2 Bachelor years

Lecture: Mo 9:45-11:15
Problem class: only at the end of term

- **0110300 Finite Element Methods**
  Instructors: Tobias Jahnke (lecture) and Benny Stein (tutorial)
  Weekly Hours: 4+2

When elliptic or parabolic differential equations on a domain with a non-trivial geometry have to be solved numerically, Finite Element Methods are often the first option. The central topic of this course is the mathematical analysis of this class of methods, in particular their construction, stability, and accuracy. The following aspects will be addressed:
- Weak formulation of elliptic boundary value problems, well-posedness
- Finite element methods for elliptic problems
- Multigrid methods
- Elliptic eigenvalue problems
- Mixed methods for saddle point problems
- Discontinuous Galerkin methods (if time permits)

This course complements the lecture Einführung in das Wissenschaftliche Rechnen (summer term 2022), which focused on practical aspects, in particular implementation. The current lecture, in contrast, will be devoted to theory, in particular well-posedness and error analysis. Both lectures are independent, i.e. Finite Element Methods can be attended without having attended Einführung in das Wissenschaftliche Rechnen. The course consists of a lecture and problem classes, both given in English.

Prerequisites: Participants are expected to have basics in numerical mathematics (methods for linear and nonlinear systems, numerical integration, interpolation etc.). Some knowledge in functional analysis (in particular Sobolev spaces) is helpful, but not a mandatory prerequisite. Programming skills are not required.

Lecture: Mo 11:30-13:00, Tu 14:00-15:30

Problem class: Wed 15:45-17:15

- **0112750 Computational Group Theory**
  Instructor Dr. Marek Kaluba
  Weekly hours: 4+2

  The course will give a gentle introduction to group theory from a computational point of view. That is not only the mathematical theory will be presented, but also the computational aspects will be considered and it is actually the latter that will determine the course of the lecture. The main aim of the course is to get acquainted with practical computability in groups, i.e. to answer broadly what kind of problems in groups could be solved in practice by computational methods.

  In the first part of the course we will do a broad overview of finite permutation groups (as: groups generated by a set of permutations), describing specific algorithms for computations inside them. The second part will be devoted to computational problems in finitely presented groups (groups defined by words over alphabet together with equations defining the relations among them). We will describe how such groups could be effectively implemented on a computer and we will take a shot at approximating the solution to the word problem.

  Tutorial classes will be mostly focused on producing practical implementations of the algorithms described in the course.

  Prerequisites: Basic concepts of group theory, basic concepts of programming and computability.

  Lecture: Th 11:30 - 13:00, Fr 09:45 - 11:15
  Problem class: Wed 14:00 - 15:30

- **0124060 Extremal Graph Theory**
  Instructor: Felix Clemen
  Weekly Hours: 2+1

  This course is an introduction to extremal graph theory, a branch of mathematics at the intersection of graph theory and extremal combinatorics, which asks questions of the following nature: How do global properties of graphs influence the local substructure? We will be taking a look at classical results in the area, such as Turán's theorem, Erdős-Stone theorem, Szemerédi's regularity lemma, and explore various techniques including probabilistic tools like the dependent random choice lemma and multistep random colorings.

Prerequisites: graph theory, combinatorics, basic probability

Lecture: Tu 9:45-11:15
Problem class: Wed 08:00-09:30

- **0155450 Introduction to Kinetic Theory**
  Instructor: Martin Frank
  Weekly hours: 2+1

  Kinetic descriptions play an important role in a variety of physical, biological, and even social applications, for instance, in the description of gases, radiations, bacteria or financial markets. Typically, these systems are described locally not by a finite set of variables but instead by a probability density describing the distribution of a microscopic state. Its evolution is typically given by an integro-differential equation. Unfortunately, the large phase space associated with the kinetic description has made simulations impractical in most settings in the past. However, recent advances in computer resources, reduced-order modeling and numerical algorithms are making accurate approximations of kinetic models more tractable, and this trend is expected to continue in the future. On the theoretical mathematical side, two rather recent Fields medals (Pierre-Louis Lions 1994, Cédric Villani 2010) also indicate the continuing interest in this field, which was already the subject of Hilbert's sixth out of the 23 problems presented at the World Congress of Mathematicians in 1900.

  This course gives an introduction to kinetic theory. Our purpose is to discuss the mathematical passage from a microscopic description of a system of particles, via a probabilistic description to a macroscopic view. This is done in a complete way for the linear case of particles that are interacting with a background medium. The nonlinear case of pairwise interacting particles is treated on a more phenomenological level.

  An extremely broad range of mathematical techniques is used in this course. Besides mathematical modeling, we make use of statistics and probability theory, ordinary differential equations, hyperbolic partial differential equations, integral equations (and thus functional analysis) and infinite-dimensional optimization. Among the astonishing discoveries of kinetic theory are the statistical interpretation of the Second Law of Thermodynamics, induced by the Boltzmann-Grad limit, and the result that the macroscopic equations describing fluid motion (namely the Euler and Navier-Stokes equations) can be inferred from abstract geometrical properties of integral scattering operators.

Prerequisites: none
Recommended courses: Partial Differential Equations, Functional Analysis

The course will be offered in flipped classroom format with lectures in video form, and tutorials/discussion sessions in presence. The discussion sessions will start in the second half of the lecture period. Exact dates will be announced.

- **0163500 Mathematical Methods in Quantum Mechanics I**
  Instructor: Dirk Hundertmark
  Weekly hours: 4+2

This course provides an introduction to the mathematical theory of (non-relativistic) quantum mechanics. This is a beautiful theory, deep in the intersection of mathematics and physics. Not only did quantum mechanics influence the development of a large part of modern mathematics, but also conversely, modern mathematics can help a lot for understanding quantum mechanics: A bit of mathematical rigor can go a long way to untangle the usual hand waving arguments in theoretical physics.

This is the second part where we will cover many–particle systems in quantum mechanics, including Coulomb systems, Fock spaces and second quantisation. In this second part we will also cover some topics of actual research. When time permits, we will also study scattering theory. The course should be attractive to math students, who want to learn about quantum mechanics, not only the basics of self-adjoint, but get really into physics stuff, and to physics students, who want to get a thorough introduction to the mathematical foundations of quantum mechanics.

Prerequisites: Basic concepts of analysis, functional analysis, and spectral theory are helpful, as is having some background in theoretical physics. However, as quoted in the introduction to the chapter "Preliminaries" in the book "Modern Methods in Mathematical Physics" by Reed and Simon: "The reader should not be discouraged if he finds that he does not have the prerequisites for reading the prerequisites" (P. Halmos).

Lecture: Tu 11:30-13:00, Th 15:45-17:15
Problem class: Fr 14:00-15:30

There are seminars available in English, please check with the instructors for a language choice.

For a current listing of courses and seminars in German and English, please see https://www.math.kit.edu/vvz/seite/vvzkommend/de For course descriptions in German, please see https://www.math.kit.edu/lehre/seite/modulhandb/de

Note that this is a preliminary schedule, the times are subject to change.