MASTER DEGREE COURSES OFFERED IN ENGLISH IN WINTER SEMESTER 2023-24

Courses are ordered by their catalog numbers:

- 0100027 Stochastic Simulation

  Instructor: Prof. Krumscheid

  Weekly Hours: 2+2

  The course covers mathematical concepts and computational tools used to analyze systems with uncertainty arising across various application domains. First, we will address stochastic modelling strategies to represent uncertainty in such systems. Then we will discuss sampling-based methods to assess uncertain system outputs via stochastic simulation techniques. The focus of this course will be on the theoretical foundations of the discussed techniques, as well as their methodological realization as efficient computational tools. Topics covered include the simulation of stochastic processes and Gaussian random fields, as well as Monte Carlo methods with a focus on variance reduction techniques and rare event simulations.

  Prerequisites: basic knowledge of Probability and Statistics, as well as of Numerical Analysis

- 0100034 Functional Data Analysis

  Instructor: Dr. Ebner

  Weekly Hours: 2+1

  The aim of the course is to give an introduction to weak convergence concepts in metric spaces and to highlight some statistical applications. We model probability measures on Borel sigma fields on infinite dimensional metric spaces and define the concept of weak convergence of probability measures. Hereby, we focus on the asymptotic behavior of sequences of random elements taking values in the space of continuous functions as well as extensions like the space of right continuous functions with existing limits from the left. Contents include the concept of weak convergence in metric spaces, Gaussian Processes, the functional central limit theorem, empirical processes, random elements in separable Hilbert spaces and applications in goodness-of-fit testing.

Prerequisites: The contents of the modules "Probability Theory" and "Mathematical Statistics" are strongly recommended.

- 0100039 Ergodic Theory

  Instructor: PD Dr. Gabriele Link

  Weekly Hours: 4+2

  A dynamical system is a set X together with a map $F\colon X \to X$ or a "flow" $\Phi\colon \mathbb{R} \times X \to X$", a conecpt we will introduce. Many problems in physics or other natural sciences (as for example describing an ideal gas or the movement of celestial bodies) consist in describing the trajectory $\{F^n(x) | n \in \mathbb{N}_0\}$ respectively $\{\Phi(t, x) \mid t \in \mathbb{R}\}$ of a point $x \in X$. This often amounts to solving complicated differential equations where in many cases a solution cannot explicitly be given. However, it is sometimes sufficient to take a statistical point of view, that is to describe the qualitative asymptotic behaviour of the trajectory of a "typical" point in X.

  In this course we will first introduce basic terminology and elementary dynamical systems such as transformations on the circle, the logistic map and Bernoulli systems. We then turn to topological dynamics, i.e. properties and phenomena for topological dynamical systems such as transitivity and recurrence. We continue with some powerful results for measurable dynamical systems, in particular Poincaré's Recurrence Theorem, von Neumann's Mean Ergodic Theorem and Birkhoff's Pointwise Ergodic Theorem. We will also introduce the notions of dynamical systems such as mixing, weak mixing and equidistribution for dynamical systems and state some results.

  As a further crucial concept in ergodic theory we define entropy, which is a measure for the complexity of a dynamical system and also relevant for example in physics and computer science.

Being familiar with the previously mentioned basic concepts and ideas of ergodic theory we will then study a more advanced topic depending on the interests of the participants of the course as for example hyperbolic dynamical systems, symbolic dynamics and coding, Furstenberg's correspondence principle.

Prerequisites: A solid background in linear algebra and analysis is indispensable. Basic knowledge in measure theory, topology, geometry, group theory and functional analysis is of advantage; however, a summary of the necessary mathematical tools from these areas will be provided during the lecture.

- 0102900 Geometric Group Theory II

  Instructor: JProf. Llosa Isenrich

  Weekly hours: 4 + 2

  Geometric Group Theory studies the interactions between finitely generated groups and geometric spaces, creating connections between algebra and geometry. In this course we will study advanced topics in Geometric Group Theory with a particular focus on hyperbolic and non-positively curved groups. Introduced by Gromov in the 1980s hyperbolic groups form an active topic of research. A hyperbolic group is a group whose Cayley graph admits a (delta-)hyperbolic metric, where we call a space hyperbolic if all of its geodesic triangles are "very thin". Gromov observed that this thin triangle condition captures many strong properties that are shared by fundamental groups of closed manifolds with negative sectional curvature (such as real hyperbolic spaces). For instance, hyperbolic groups are always finitely presented, they do not contain higher rank free abelian subgroups, they have solvable word problem and they satisfy a strong version of the so-called Tits alternative. In the first part of the course we will introduce hyperbolic groups and spaces and then show that they satisfy all of these strong properties. In the second half we then plan to move on to other advanced topics in Geometric Group Theory, such as more general non-positively curved groups.

  Prerequisites: Basic knowledge in Geometric Group Theory and Elementary Geometry and Topology (such knowledge is e.g. provided by the KIT courses Geometric Group Theory I and "Elementare Geometrie").

- 0104500 Graph Theory

  Instructor: Prof. Maria Axenovich

  Weekly Hours: 4+2

  Graphs are structures in discrete mathematics that in particular model various networks. The course starts with basic concepts in graph theory such as trees, cycles, matchings, factors, connectivity, and their interconnections. Further topics include properties of graphs with forbidden subgraphs, planar graphs, graph colorings, random graphs, Ramsey theory, and graph minors. Not only classical, but very recent results in the field will be discussed. The class is oriented towards problem solving. Particular attention to proof writing techniques will be paid in the problem class. The final grade will be based on the written exam. Bonus points will be given for weekly or biweekly homework assignments.

  Prerequisites: Linear algebra and basic probability theory

- 0105600 Spatial Stochastics

  Instructor: Prof. Daniel Hug

  Weekly Hours: 4+2

  The lectures provide an introduction to fundamental subjects involving spatial stochastic processes. We will discuss general concepts such as point processes and random measures and describe some of their properties. In addition, we will study specific models relevant for applications. These include Poisson processes and Gaussian random fields which are highly relevant in natural and earth sciences as well as in engineering (telecommunication, materials science). The following topics will be covered: Point processes, random measures, Poisson processes, Gibbs point processes, Palm distributions, spatial ergodic theorems, spectral theory of random fields, Gaussian fields.

Competence Goals: The students are familiar with some basic spatial stochastic processes. They do not only understand how to deal with general properties of distributions, but also know how to describe and apply specific models (Poisson process, Gaussian random fields). They know how to work self-organised and self-reflexive.

Recommendations (expected prior knowledge): Basic concepts of measure and integration theory, probability theory

- 0105900 Evolution Equations

  Instructor: Prof. Schnaubelt

  Weekly hours: 4+2

  Evolution equations describe the time evolution of dynamical systems by an ordinary differential equation in a Banach space. We investigate linear and autonomous (time invariant) problems. In this case the solutions are given by a one-parameter semigroup of linear operators. For such operator semigroups there is a quite complete theory, which allows us to study the properties of the underlying dynamical system. This approach essentially relies on functional analytic methods and results.

  We treat the basic existence theorems for linear autonomous evolution equations. In this framework, we then investigate qualitative properties of the solutions, such as regularity and the longterm behavior. Perturbation and approximation results are also studied (which have connections to numerical analysis). The developed theory can be applied to the diffusion, the (damped) wave, and the Schrödinger equation.

  Prerequisites: Functional analysis and theory of L^p spaces. The necessary parts from the lecture Spectral Theory will be recalled (without proofs) and discussed.

- 0107800 Numerical methods in mathematical finance

  Instructor: Prof. Tobias Jahnke

  Weekly Hours: 4+2

An option is a contract which gives its owner the right to buy or sell an underlying asset at a future time at a fixed price. The underlying asset is often a stock of a company, and since its value varies randomly, computing the fair price of the corresponding option is an important and interesting problem which yields a number of mathematical challenges. This lecture provides an introduction to a number of models for option pricing. The main goal, however, is the construction and analysis of numerical methods which approximate the solution of the corresponding differential equations in a stable, accurate and efficient way. The following topics will be treated:

* Options, arbitrage and other basic concepts
* Black-Scholes equation und Black-Scholes formulas
* Numerical methods for stochastic differential equations
* (Multilevel) Monte Carlo methods
* (Quasi-)Monte Carlo integration
* Numerical methods for Black-Scholes equations
* Numerical methods for American options

Prerequisites: Participants are expected to be familiar with stochastic differential equations, the Ito integral, and the Ito formula. MATLAB skills are strongly recommended for the programming exercises.

- 0109400 Mathematical Modelling and Simulation

  Instructor: PD Dr. Thäter

  Weekly Hours: 2+1

  This course is open to mathematicians and engineers. The general aim of this lecture course is threefold:

  1) to interconnect different mathematical fields
  2) to connect mathematics and real life problems
  3) to learn to be critical and to ask relevant questions.

  During the lecture course there will be one lecture of a person from industry and one excursion (at the end of the lecture course).

  To earn the credits you have to attend the lecture, finish the work on one project during the term in a group of 2-3 persons and pass the exam.

The topic of the project is up to the choice of each group.

Prerequisites: Basic mathematical concepts from the first 2 Bachelor years

Problem class only at the end of term
Please register with the ILIAS-course before the term starts.

- 0155450 Introduction to Kinetic Theory

  Instructor: Prof. Frank

  Weekly hours: 2+1 (The course will be offered in flipped classroom format. Lecture videos are combined with tutorials/discussion sessions in presence).

  Kinetic descriptions play an important role in a variety of physical, biological, and even social applications, for instance, in the description of gases, radiations, bacteria or financial markets. Typically, these systems are described locally not by a finite set of variables but instead by a probability density describing the distribution of a microscopic state. Its evolution is typically given by an integro-differential equation. Unfortunately, the large phase space associated with the kinetic description has made simulations impractical in most settings in the past. However, recent advances in computer resources, reduced-order modeling and numerical algorithms are making accurate approximations of kinetic models more tractable, and this trend is expected to continue in the future. On the theoretical mathematical side, two rather recent Fields medals (Pierre-Louis Lions 1994, Cédric Villani 2010) also indicate the continuing interest in this field, which was already the subject of Hilbert's sixth out of the 23 problems presented at the World Congress of Mathematicians in 1900.

  This course gives an introduction to kinetic theory. Our purpose is to discuss the mathematical passage from a microscopic description of a system of particles, via a probabilistic description to a macroscopic view. This is done in a complete way for the linear case of particles that are interacting with a background medium. The nonlinear case of pairwise interacting particles is treated on a more phenomenological level.

An extremely broad range of mathematical techniques is used in this course. Besides mathematical modeling, we make use of statistics and probability theory, ordinary differential equations, hyperbolic partial differential equations, integral equations (and thus functional analysis) and infinite-dimensional optimization. Among the astonishing discoveries of kinetic theory are the statistical interpretation of the Second Law of Thermodynamics, induced by the Boltzmann-Grad limit, and the result that the macroscopic equations describing fluid motion (namely the Euler and Navier-Stokes equations) can be inferred from abstract geometrical properties of integral scattering operators.

Prerequisites: none
Recommended courses: Partial Differential Equations, Functional Analysis

- 105964 Introduction to Convex Integration

  Instructor: Dr. Zillinger

  Weekly Hours: 2+0

  This lecture provides an introduction to the methods of convex integration and their applications. In this semester there will be specific focus on:
  - shape-memory alloys
  - elasticity, stress-free states and differential inclusions
  - rigidity and notions of convexity
  - convex integration constructions, surface energies and higher regularity

  Prerequisites: Basic concepts of partial differential equations and functional analysis

- 105966 Space and time discretisation of nonlinear wave equations

  Instructor: Prof. Marlis Hochbruck and Benjamin Dörich

  Weekly hours: 3+1

  The aim of this lecture is the error analysis of space and time discretizations of nonlinear wave equations. We discuss evolutions equations with monotone operators on Hilbert spaces and different kinds of spatial discretizations, e.g., finite element methods, discontinuous Galerkin methods or spectral methods. Afterwards, we study their combination with time integration schemes like the Crank-Nicolson method and an implicit-explicit scheme. The abstract analysis is illustrated with concrete examples.

Prerequisites: The students are expected to be familiar with the basics of the numerical analysis of the time integration of ordinary differential equations (Runge-Kutta and multistep methods) and of finite element methods for elliptic boundary element methods. Some basic knowledge in functional analysis and the analysis of boundary value problem is helpful but the main results will be repeated in the lecture.

- Introduction to dynamical systems

  Instructor: Dr. Björn de Rijk

  Weekly Hours: 3+1

  A dynamical system consists of a state space and a dynamical rule describing the time evolution of points in the state space, i.e., what future states follow from the current state. In this course we focus on continuous, or differential, dynamical systems, where the dynamical rule is given by an ordinary (or partial) differential equation. Such systems form the basis of physical models that exhibit smooth change and naturally arise in many scientific disciplines such as physics, biology, chemistry and engineering. Rather than calculating explicit solutions (which are known in only very few examples), we develop analytical and geometrical techniques to study the qualitative properties of dynamical systems. In particular, we treat the following concepts:

  * Flows
  * Abstract dynamical systems
  * Lyapunov functions
  * Invariant sets
  * Limit sets and attractors
  * Hartman-Grobman theorem
  * Local (un)stable manifold theorem
  * Poincaré-Bendixson theorem
  * Periodic orbits and Floquet theory
  * Exponential dichotomies
  * Melnikov functions

* Lin's method
* Hamiltonian dynamics
* Liénard systems
* Bifurcations
* Chaotic dynamics
* (Introduction to) Fenichel theory
* Center manifolds
* Dynamical systems associated with semilinear evolution equations

Prerequisites: ordinary differential equations and linear algebra

There are seminars available in English, please check with the instructors for a language choice. Here are just some of them listed:

- 0102650 Seminar (Statistical Forecasting and Classification)

  Instructor: Prof. Gneiting

  Weekly Hours: 2

  A common desire of all humankind is to make predictions for an uncertain future. Clearly then, forecasts ought to be probabilistic, i.e., they ought to take the form of probability distributions over future quantities or events. In this seminar, we will study advanced facets of the probabilistic and statistical foundations of forecasting and classification problems.

  Prerequisites: Prerequisites include an introductory course in probability and statistics ("Einführung in die Stochastik" or equivalent) and an advanced course in probability and measure ("Wahrscheinlichkeitstheorie" or equivalent). Successful completion of the course sequence "Forecasting: Theory and Practice I and II" is expected and strongly recommended. In particular, participants need to be familiar and confident with the contents of the papers by Gneiting and Ranjan (Electronic Journal of Statistics, 2013) and Gneiting and Katzfuss (Annual Review of Statistics and Its Application, 2014).