INTERNATIONAL PROGRAM (MASTER)

CLASSES: WINTER SEMESTER 2009/2010

1042 Riemannian Geometry Prof. Enrico Leuzinger
Lecture: 4 h, 8 credit points;
Wed 08:00-09:30 Kl.ETI Geb11.10, Thu 08:00-09:30 AOC 101 Geb30.45
Tutorial: 2 h, 2 credit points;
Fr 08:00-09:30 1C-04 Geb05.20

1046 Partial Differential Equations Dr. Matthias Kurzke
Lecture: 4 h, 8 credit points;
Tue 11:30-13:00 Kl.HS Geb10.50, Wed 14:00-15:30 Eiermann
Tutorial: 2h, 2 credit points;
Mo 15:45-17:15 Hertz

1048 Functional Analysis Prof. Roland Schnaubelt
Lecture: 4 h, 8 credit points;
Tue 09:45-11:15 Nusselt, Thu 11:30-13:00 Hertz
Tutorial: 2h, 2 credit points;
Fr 09:45-11:15 NH Geb20.40

1054 Variational Methods and Applications to PDEs
Prof. Michael Plum & Prof. Wolfgang Reichel
Lecture: 2 h, 4 credit points;
Mo 14:00-15:30 S 33
Tutorial: 1 h, 1 credit point;
Tue 15:45-17:15 S 33

1080 Iterative Methods for Spare Linear Systems Dr. Jan Mayer
Lecture: 2 h, 4 credit points;
Fr 15:45-17:15 1C-03 Geb05.20

1081 Time Series Analysis Prof. Claudia Kirch
Lecture: 4 h, 8 credit points;
Mo 09:45-11:15 HS 101 Geb10.50, Wed 11:30-13:00 702 Geb10.50
Tutorial: 2 h, 2 credit points;
Fr 11:30-13:00 HS 101 Geb10.50

1097 High Dimensional Approximation Prof. Tobias Jahnke
Lecture: 2 h, 4 credit points;
Tue 08:00-09:30 1C-04 Geb05.20

1246 Seminar: Boundary and Eigenvalue Problems Prof. Michael Plum
Thu 14:00-15:30 1C-01 Geb05.20

1254 Seminar: Engineering Mathematics & Computing Prof. Vincent Heuveline
Thu 15:45-17:15

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30		High dim	Riemannian	Riemannian	Riemannian (T)
09:45-11:15	Time Series	FA			FA (T)
11:30-13:00		PDE	Time Series	FA	Time Series (T)
14:00-15:30	Variational		PDE		
15:45-17:15	PDE (T)	Variational (T)			Iterative

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: WINTER SEMESTER 2009/2010

1042 Riemannian Geometry Prof. Enrico Leuzinger
Lecture: 4 h, 8 credit points;
Wed 08:00-09:30 Kl.ETI Geb11.10, Thu 08:00-09:30 AOC 101 Geb30.45
Tutorial: 2 h, 2 credit points;
Fr 08:00-09:30 1C-04 Geb05.20

Contents:
We first explain what an n-dimensional, differentiable manifold is. This is one of the fundamental concepts of modern mathematics and physics (e.g. important in Einstein's theory of relativity). It is designed in order to extend and to apply analytic tools to more abstract spaces. In a Riemannian manifold every tangent space is equipped with an scalar product. Using this additional structure it is possible to widely generalize concepts from the intrinsic geometry of surfaces: covariant derivative, parallel translation, geodesics, curvature, Jacobi-fields. Eventually we will use all these devices to investigate global questions about manifolds (in particular the relation between curvature and topology).

Textbooks:
Manfredo P. do Carmo, Riemannian Geometry, Birkhäuser, 1972.
M. Spivak, A Comprehensive Introduction to Differential Geometry, 5 vols, Publish or perish, 1972.

Prerequisites:
Basic concepts of analysis and topology; a course on the differential geometry of surfaces might be helpful.

1046 Partial Differential Equations Dr. Matthias Kurzke
Lecture: 4 h, 8 credit points;
Tue 11:30-13:00 Kl.HS Geb10.50, Wed 14:00-15:30 Eiermann
Tutorial: 2h, 2 credit points;
Mo 15:45-17:15 Hertz

Partial differential equations (PDE) are a wide subject, with examples and applications coming from many areas of mathematics, science, and engineering. In this lecture, we will concentrate on some exemplary classes of PDE, and study methods to solve them (or at least prove the existence of solutions) as well as the properties of these solutions. A special focus will be on Laplace's equation, whose solutions are called harmonic functions and have many interesting properties, and its dynamic counterparts, the heat equation and the wave equation. Some more general classes of linear PDE and special examples of nonlinear PDE will also be studied.

Prerequisites for the course are a knowledge of real analysis including multivariable calculus and the integral theorems, the basic theory of ordinary differential equations and a knowledge of linear algebra concepts, on the level of the lectures on Analysis 1-3 and Linear Algebra 1.

Literature:
EVANS, L. C. Partial Differential Equations. AMS Graduate Studies in Mathematics 19, Providence, RI 1998.
HAN, Q., LIN, F. Elliptic Partial Differential Equations. Courant Lecture Notes 1. New York, 1997. Reprinted by the AMS, Providence, RI 2000.
JOHN, F. Partial Differential Equations. 4th edition. Applied Mathematical Sciences 1, Springer, New York 1982.
JOST, J. Partial Differential Equations. Graduate Texts in Mathematics 214. Springer, New York 2002.

1048 Functional Analysis Prof. Roland Schnaubelt
Lecture: 4 h, 8 credit points;
Tue 09:45-11:15 Nusselt, Thu 11:30-13:00 Hertz
Tutorial: 2h, 2 credit points;
Fr 09:45-11:15 NH Geb20.40

Contents:
The lecture is concerned with Banach and Hilbert spaces as well as linear operators acting on these spaces. Typical examples are spaces of continuous and integrable functions and linear maps, which one defines via integration of such functions. In this way one can formulate integral equations as affine or linear equations on a suitable Banach space, and one can solve them by means of functional analytic methods. This class of problems was in fact the historical starting point for the development of functional analysis around 1900. In the following years it became a fundamental area of modern analysis and its applications in- and outside of mathematics. A preliminary list of topics:

1. basic properties and examples of metric and Banach spaces and of linear operators,
2. Hilbert spaces (scalar product, orthogonal projection and basis),
3. principle of uniform boundedness and open mapping theorem

4. dual spaces and theorem of Hahn-Banach,
5. weak convergence and theorem of Banach-Alaouglu,
6. Fourier transform and applications to partial differential equations,
7. spectral theorem for compact and self adjoint Operators.

Prerequisites: Analysis 1-3 and Linear Algebra 1+2.

References:

1. H.W. Alt: *Lineare Funktionalanalysis.* Springer.
2. J.B. Conway: *A Course in Functional Analysis.* Springer.
3. M. Schechter: *Principles of Functional Analysis.* Academic Press.
4. A.E. Taylor, D.C. Lay: *Introduction to Functional Analysis.* Wiley.
5. D. Werner: *Funktionalanalysis.* Springer.

1054 Variational Methods and Applications to PDEs
Prof. Michael Plum & Prof. Wolfgang Reichel
Lecture: 2 h, 4 credit points;
Mo 14:00-15:30 S 33
Tutorial: 1 h, 1 credit point;
Tue 15:45-17:15 S 33

Contents: We will consider functionals defined on Banach-spaces and find conditions, such that these functionals possess minimizers or – more generally – critical points. Sometimes such minimizers have physical significance, e.g., they may represent energetically optimal configurations in material sience (e.g. soap bubbles, buckling plates or beams, orientation of liquid crystals under a magnetic force). A neccessary condition for a minimizer is that it has to satisfy the **Euler-Lagrange equation** (corresponding to the vanishing of the first derivative of a real valued function at a local minimum or local maximum). Often the Euler-Lagrange equation is a nonlinear elliptic partial differential equation. In this lecture we will focus on applying the calculus of variations as a tool to provide existence of solutions to nonlinear elliptic partial differential equations.

1. weak convergence, lower-semicontinuity, convexity
2. first variation, Euler-Lagrange equation, Gateaux- and Fréchet-differentiability
3. Sobolev spaces, weak solutions of elliptic PDEs
4. constraint optimisation, Lagrange multipliers
5. saddle points, mountain-pass lemma

Wherever possible, we will complement the above topics with examples from elliptic partial differential equations.

Prerequisites:
Multi-variable calculus, functional analysis. A background in partial differential equations is not neccessary, but helpful. The lecture is suitable for students in mathematics, physics and engineering.

Literature:
Giaquinta, Hildebrandt: Calculus of Variations I, Springer 1996
Struwe: Variational Methods, Springer 1998
Willem: Minimax theorems, Birkhäuser, 1997

1080 Iterative Methods for Spare Linear Systems Dr. Jan Mayer
Lecture: 2 h, 4 credit points;
Fr 15:45-17:15 1C-03 Geb05.20

Numerical algorithms for finding an approximate solution to some real life problems often yield linear systems having only a few non-zero coefficients in each equation. Such systems are called sparse. They occur in particular for problems having 2D or 3D geometry and whose solution can be described by partial differential equations. The variables describing an approximation of the solution usually depend only on variables that are "close" in the underlying geometry, but not on any variables that are farther away. As a consequence, only very few variables actually occur in any linear equation. Similarly, any problem that can be described using graphs (e.g. networks, circuits) often results in sparse linear systems. More specifically, sparse linear systems occur in electrical circuit and semiconductor device simulation, fluid dynamics problems, chemical process simulation, macroeconomic modeling and many other applications.

As the coefficient matrix of these systems is large and sparse, typically having no more than 2-100 non-zero elements per row on average and generally having a dimension of at least 100,000 and often much more, simply using Gaussian elimination to solve the system is not feasible. Gaussian elimination does not exploit the large number of zeroes, so that computation time is generally unacceptably high. More importantly, computers often do not have the memory needed to store an entire matrix of this size. Instead, it is usually only possible to store the non-zero elements. As a consequence, we need algorithms for solving a linear system which only make use of these non-zero elements. Iterative methods, requiring only matrix-vector-multiplications with the coefficient matrix, satisfy this condition because all zero elements can be ignored for calculating matrix-vector-products.

In this course, we will cover different iterative methods for solving sparse linear systems. After considering some simple approaches, such as the minimal residual or steepest descent methods, we will discuss more sophisticated projection methods with a special focus on Krylov subspace methods such as GMRES, CG and BiCG. Special emphasis will be placed on the advantages and disadvantages of each method and on convergence.

The course requires a good working background in linear algebra. Previous experience with numerical mathematics is helpful but not essential.

Textbooks:
Axelson, Owe: Iterative Solution Methods.
Meurant, Gerard: Computer Solution Methods for Large Linear Systems.
Saad, Yousef: Iterative Methods for Sparse Linear Systems, 2nd Edition.
van der Vorst, Henk: Iterative Krylov Methods for Large Linear Systems.

1081 Time Series Analysis Prof. Claudia Kirch
Lecture: 4 h, 8 credit points;
Mo 09:45-11:15 HS 101 Geb10.50, Wed 11:30-13:00 702 Geb10.50
Tutorial: 2 h, 2 credit points;
Fr 11:30-13:00 HS 101 Geb10.50

Requirements
Fundamental knowledge of probability theory, e.g. 'Stochastik 2'. Some basic knowledge in statistics is helpful.

Content
The lecture gives an introduction to time series analysis. Topics that are dealt with:

- Stationary Time Series
  - Trend and seasonal components
  - Autocorrelation and spectral measure
  - ARMA-models
  - Wold decomposition
- Statistic in the time domain of stationary processes
  - Prediction
  - Estimators for mean and autocorrelation function
  - Estimation in ARMA models
- Statistic in the frequency domain of stationary processes
  - The periodogram
  - Nonparametric spectral density estimators
- Extensions and financial time series
  - ARIMA and SARIMA models
  - GARCH models

If time allows, we will also discuss further topics such as unit-root-tests or multivariate time series.

References

- Brockwell, Davis: Time Series: Theory and Methods

- Kreiß, Neuhaus: Einführung in die Zeitreihenanalyse
- Franke, Härdle, Hafner: Statistics of financial markets (dt. Ausgabe: Einführung in die Statistik der Finanzmärkte)

1097 High Dimensional Approximation Prof. Tobias Jahnke
Lecture: 2 h, 4 credit points;
Tue 08:00-09:30 1C-04 Geb05.20

Many important applications in financial mathematics, systems biology, chemistry, or physics require the solution of high-dimensional partial differential equations. Such problems are particularly challenging because they cannot be solved with traditional methods. The main reason is that the number of unknowns grows exponentially with respect to the dimension such that the computational workload exceeds the capacity of most computers. For example, an equidistant discretization of the unit interval $[0, 1]$ by mesh points with distance 0.1 has only 11 points $(0, 0.1, ...0.9, 1)$, but a similar discretization of the unit cube requires 113 = 1331 mesh points, and a corresponding mesh on the 10-dimensional hypercube contains $11^{10} = 25,937,424,601$ mesh points. This exponential growth of the size of the problem is known as *the curse of dimesionality*.

In this lecture, we will give examples for applications which lead to high-dimensional problems and discuss the properties of the corresponding equations (Master equation, Fokker-Planck equation, Schrödinger equation). Then, three strategies to avoid the curse of dimensionality will be introduced and analyzed: sparse grids, wavelet compression, and variational approximation. Special emphasis will be devoted to the question *why these approaches work* and *which assumptions have to be made*. The main goal of this lecture, however, is to convince the audience week by week that it pays to get up early to attend a lecture at 8 a.m.

The lecture will be given in English. It will be suited for students in mathematics, physics and other sciences with a basic knowledge in ordinary and partial differential equations and the corresponding numerical methods.

Literature:

- H.-J. Bungartz and M. Griebel.
  Sparse grids.
  Acta Numerica, 13:1-123, 2004.
  `http://wissrech.ins.uni-bonn.de/research/pub/griebel/sparsegrids.pdf`
- A. Cohen, W. Dahmen, and R. DeVore.
  Adaptive wavelet methods for elliptic operator equations: Convergence rates.
  Math. Comput. 70(233):27-75, 2001.
  `http://www.ams.org/mcom/2001-70-233/S0025-5718-00-01252-7/home.html`
- Christian Lubich.
  From quantum to classical molecular dynamics: Reduced models and numerical analysis.
  Zurich Lectures in Advanced Mathematics. Zürich: European Mathematical Society (EMS), 2008.

Contact: JProf. Dr. Tobias Jahnke
E-mail: tobias.jahnke@kit.edu
Office 4C-11, Geb. 05.20 (Allianz building)
Phone: 608 7982
http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/