INTERNATIONAL PROGRAM (MASTER)

CLASSES: WINTER SEMESTER 2010/2011

1044 Convex Geometry
Lecture: 4 h, 8 credit points
Mon 11:30-13:00 AOC 201 Geb 30.45, Tue 11:30-13:00 1C-04 Geb 5.20
Tutorial: 2 h, 2 credit points
Wed 14:00-15:30 1C-03 Geb 5.20
apl. Prof. Dr. Daniel Hug, Dipl.-Math. oec. Sven Ebert

1048 Functional Analysis
Lecture: 4 h, 8 credit points
Tue 9:45-11:15 Nusselt Geb 10.23, Thu 11:30-13:00 Hertz Geb 10.11
Tutorial: 2h, 2 credit points
Fri 14:00-15:30 Eiermann Geb 20.40
Prof. NN

1053 Nonlinear Schrödinger Equations - stationary aspects
Lecture: 2 h, 4 credit points
Thu 11:30-13:00 1C-04 Geb 5.20
Prof. Wolfgang Reichel

1054 Nonlinear Boundary Value Problems
Lecture: 4 h, 8 credit points
Mon 11:30-13:00 1C-01 Geb 5.20, Fri 9:45-11:15 Z 1 Geb 1.85
Tutorial: 2 h, 2 credit points
Wed 15:45-17:15 1C-01 Geb 5.20
Prof. Michael Plum

1059 Evolution Equations
Lecture: 4 h, 8 credit points
Wed 8:00-9:30 1C-01 Geb 5.20, Thu 8:00-9:30 1C-01 Geb 5.20
Tutorial: 2 h, 2 credit points
Mon 14:00-15:30 1C-04 Geb 5.20
Prof. Roland Schnaubelt

1065 Applied Bayesian Inference A
Lecture: 2 h, 4 credit points
20.10.-15.12.2010: Wed 9:45-11:15 1C-04 Geb 5.20;
21.10.-16.12.2010: Thu 9:45-11:15 1C-04 Geb 5.20
Tutorial: 2 h, 2 credit points
22.10.-17.12.2010: Fri 11:30-13:00 1C-04 Geb 5.20
Prof. Renate Meyer

1102 Exponential Integrators
Lecture: 2 h, 4 credit points
Wed 11:30-13:00 1C-03 Geb 05.20
Tutorial: 1 h, 2 credit points
Thu 15:45-17:15, every two weeks, Z 1 Geb 1.85
Prof. Marlies Hochbruck

1235 Seminar: Boundary value Problems
Wed 11:30-13:00 Z 2 Geb 1.85
Prof. Wolfgang Reichel

1254 Seminar: Engineering Mathematics and Computing
Thu 14:00-15:30 R NN Geb NN
Prof. Vincent Heuveline, Dr. Rudi Klatte

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30			Ev.Equ.	Ev.Equ.	
09:45-11:15		Func.A.	Appl.Bay.	Appl.Bay.	Nonlin. BVP
11:30-13:00	Conv.Geo. Nonlin. BVP	Conv.Geo.	Exp.Integr.	Func.A. Nonl. Schr.	
14:00-15:30					
15:45-17:15					

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: WINTER SEMESTER 2010/2011

1044 Convex Geometry

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Contents
Convexity is a fundamental notion in mathematics which has a combinatorial, an analytic, a geometric and a probabilistic flavour. Basically, a given set A in a real vector space is called convex if with any two points of A the segment joining the two points is also contained in A. This course provides an introduction to the geometry of convex sets in a finite-dimensional real vector space.

The following topics will be covered:

1. Geometric foundations: combinatorial properties, support and separation theorems, extremal representations

2. Convex functions

3. The Brunn-Minkowski Theory: basic functionals of convex bodies, mixed volumes, geometric (isoperimetric) inequalities

4. Integral geometric formulas

If time permits, we also consider additional topics such as symmetrization of convex sets or sets of constant width.

Prerequisites
This course is suited for everybody with a firm background in analysis and linear algebra.

References

Gruber, P.M. Convex and Discrete Geometry. Grundlehren 336, Springer, 2007.

Schneider, R. Convex Bodies: The Brunn-Minkowski Theory. Encyclopedia of Mathematics and its Applications. Cambridge University Press, Cambridge, 1993.

Lecture Notes will be provided as the course proceeds.

1048 Functional Analysis
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Further information will follow.

Below is some preliminary information based on the course from last year.

Contents

The lecture is concerned with Banach and Hilbert spaces as well as linear operators acting on these spaces. Typical examples are spaces of continuous and integrable functions and linear maps, which one defines via integration of such functions. In this way one can formulate integral equations as affine or linear equations on a suitable Banach space, and one can solve them by means of functional analytic methods. This class of problems was in fact the historical starting point for the development of functional analysis around 1900. In the following years it became a fundamental area of modern analysis and its applications in- and outside of mathematics.

A preliminary list of topics:

1. Basic properties and examples of metric and Banach spaces and of linear operators
2. Hilbert spaces (scalar product, orthogonal projection and basis)
3. Principle of uniform boundedness and open mapping theorem
4. Dual spaces and theorem of Hahn-Banach
5. Weak convergence and theorem of Banach-Alaouglu
6. Fourier transform and applications to partial differential equations
7. Spectral theorem for compact and self adjoint Operators.

Prerequisites
Analysis 1-3 and Linear Algebra 1+2.

References

H.W. Alt: Lineare Funktionalanalysis. Springer.

J.B. Conway: A Course in Functional Analysis. Springer.

M. Schechter: Principles of Functional Analysis. Academic Press.

A.E. Taylor, D.C. Lay: Introduction to Functional Analysis. Wiley.

D. Werner: Funktionalanalysis. Springer.

1053 Nonlinear Schrödinger Equations - stationary aspects

Lecture: 2 h, 4 credit points
Thu 11:30-13:00 1C-04 Geb 5.20
Prof. Wolfgang Reichel

Contents

This course is a continuation of the lecture of Prof. Schnaubelt in the previous summer semester on "Nonlinear Schrödinger Equations - dynamical aspects". However, the contents and methods are independent.
In this course I will study solitary waves of the nonlinear Schrödinger equation (NLS). They are solutions of

$$-\Delta u + V(x)u = \Gamma(x)(u)^{p-1}u \text{ in } \mathbb{R}^n.$$

I will mainly discuss variational methods for proving existence of solutions. In the case of constant coefficients we will also study qualitative properties of positive solutions.
A preliminary list of topics:

- Motivation and examples
- Constant coefficient case
- Asymptotically constant coefficients
- Periodic coefficients

Prerequisites

Familiarity with variational methods is helpful. Some of the basic facts of the calculus of variations will be reviewed. Knowledge in Lebesgue integral, Sobolev spaces, and functional analytical concepts like weak convergence is essential.

References

H. Berestycki, P.L. Lions: Nonlinear scalar field equations I. Arch. Rational Mech. Anal. 82, 313-345 (1983).

B. Gidas, Wei-Ming Ni, L. Nirenberg: Symmetry of positive solutions of nonlinear elliptic equations in $\mathbb{R}^n$. Math. Anal. Appl., Part A, 369-402. Adv. in Math. Suppl. Stud. 7a (1981).

A. Pankov: Periodic nonlinear Schrödinger equation with application to photonic crystals. Milan J. Math. 73, 259-287 (2005).

W. Strauss: Existence of solitary waves in higher dimensions. Comm. Math. Phys. 55, 149-162 (1977).

M. Struwe: Variational Methods. Springer Verlag.

C. A. Stuart: A variational approach to bifurcation in L^P on an unbounded symmetrical domain. Math. Ann. 263, 51-59 (1983).

M. Willem: Minimax Theorems. Birkhäuser Verlag.

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Prof. Michael Plum

Further information will follow.

1059 Evolution Equations
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Wed 8:00-9:30 1C-01 Geb 5.20, Thu 8:00-9:30 1C-01 Geb 5.20
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Prof. Roland Schnaubelt

Contents

Evolution equations describe the time evolution of dynamical systems by an ordinary differential equation in a Banach space. We first investigate linear and autonomous (time invariant) problems. Based on this theory, we then consider nonlinear (more precisely, semilinear) equations. The solutions in the linear case are represented by a one-parameter semigroup of linear operators. In the semilinear case, the solution is given by the formula of variation of constants. For such operator semigroups there is a quite complete theory, which allows to study the properties of the underlying dynamical system. This approach essentially relies on functional analytic methods and results. We treat the basic existence theorem for linear and semilinear autonomous evolution equations. In this framework, we then investigate qualitative properties of the solutions (e.g., the longterm behavior). These results can be applied to the diffusion, the wave and the (nonlinear) Schrödinger equation.

Prerequisites

Functional analysis. The necessary parts from the lecture Spectral Theory will be recalled (without proofs).

References

K.-J. Engel and R. Nagel, One-Parameter Semigroups for Linear Evolution Equations. Springer, 2000.

A. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations. Springer, 1983.

1065 Applied Bayesian Inference A
Lecture: 2 h, 4 credit points
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Prof. Renate Meyer

Contents
This is an introductory course in Bayesian inference starting from first principles. The Bayesian approach is based on a different paradigm than the classical frequentist approach to statistical inference. Over the last decade, the Bayesian approach has revolutionised many areas of applied statistics such as biometrics, econometrics, market research, statistical ecology and physics. Although the Bayesian approach dates back to the 18th century, its rise and enormous popularity today is due to the advances made in Bayesian computation through computer-intensive simulation methods. Knowledge of Bayesian procedures and software packages will become indispensable for any career in Statistics. Students will be using the software package R for Bayesian computation and will be introduced to WinBUGS. Topics covered include: the Bayesian approach, conjugate distributions, prior distributions, simulation methods, Markov chain Monte Carlo methods, using the WinBUGS software, and applications to data analysis.

Prerequisites
Basic knowledge in probability and statistics; Familiarity with the classical frequentist approach to statistical inference and knowledge of SPLUS or R is of advantage.

References

Textbooks: Recommended for complementary reading:

Berger, J.O., Wolpert R.L.(1988): The Likelihood Principle, IMS Lecture Notes-Monograph Series

Carlin,B.P. andTh.A.Louis(2000): Bayes and Empirical Bayes Methods for Data Analysis, Chapman and Hall, London

Congdon, P.(2001): Bayesian Statistical Modelling, Wiley, New York

Gelman, A., J.B. Carlin, H.S. Stern, D.B. Rubin(2004): Bayesian Data Analysis, Chapman and Hall, London.

Lee, P.M.(2004): Bayesian Statistics: An Introduction, Halsted Press, New York.

Albert, J.(2007): Bayesian Computation with R, Springer, New York.

Assignment questions, solutions and other handouts will be distributed in lectures. After class extra copies can be obtained from the lecturer's office or the course webpage.

Course Outline

1. Introduction to R and WinBUGS

2. Introduction to the Bayesian approach
(a) History of statistical inference
(b) Bayes' theorem: discrete case
(c) Likelihood-based functions
(d) Bayes' theorem: continuous case
(e) Conjugate examples: Bernoulli, Binomial, Uniform, Normal, Poisson, and Exponential
(f) Exchangeability
(g) Sequential Learning
(h) Comparing Bayesian and Frequentist Inference for a Proportion

3. Fundamental Principles
(a) Likelihood Principle
(b) Conditionality Principle
(c) Sufficiency Principle
(d) Stopping Rule Principle

4. Prior Distributions
(a) Subjective priors
(b) Conjugate priors
(c) Noninformative priors
(d) Jeffreys priors
(e) Empirical Bayes priors
(f) Hierarchical priors

In case the course will be offered in two parts, the preceding chapters 1-4 will be covered in the 1-week block course before the begin of the winter semester while the following chapters 5-7 will be taught in a regular(2+1) classduring the semester.

5. Posterior Computation
(a) Numerical integration
(b) Asymptotic approximations
(c) Non-iterative simulation: inverse transform and rejection method
(d) Stochastic Simulation: rejection and SIR
(e) MarkovChainMonteCarlo(MCMC) Methods
(f) Metropolis-Hastings algorithm
(g) Practical implementation issues
(h) Markov chain theory
(i) Gibbs sampler
(j) Adaptive Rejection Sampling

6. Software: WinBUGS
(a) Introduction
(b) Linear Models and applications
(c) Generalized Linear Models and applications
(d) Dynamic Models and applications
(e) Convergence Diagnostics using CODA

7. Model Checking
(a) Bayes factors
(b) Bayesian p-values
(c) Bayesiandeviance(DIC)

1102 Exponential Integrators

Lecture: 2 h, 4 credit points
Wed 11:30-13:00 1C-03 Geb 05.20
Tutorial: 1 h, 2 credit points
Thu 15:45-17:15, every two weeks, Z 1 Geb 1.85
Prof. Marlies Hochbruck

Contents

In this class we consider the construction, analysis, implementation and application of exponential integrators. The focus will be on two types of stiff problems. The first one is characterized by a Jacobian that possesses eigenvalues with large negative real parts. Parabolic partial differential equations and their spatial discretization are typical examples. The second class consists of highly oscillatory problems with purely imaginary eigenvalues of large modulus. Apart from motivating the construction of exponential integrators for various classes of problems, our main intention in this class is to present the mathematics behind these methods. We will derive error bounds that are independent of stiffness or highest frequencies in the system. Since the implementation of exponential integrators requires the evaluation of the product of a matrix function with a vector, we will briefly discuss some possible approaches as well.

Prerequistes

Basic knowledge in numerical methods for ordinary differential equations.

References

M. Hochbruck and A. Ostermann, Exponential Integrators, Acta Numerica, vol. 19, pp. 209-286 (2010)