INTERNATIONAL PROGRAM (MASTER)

CLASSES: SUMMER SEMESTER 2014

0161300 Numerical Analysis of highly-oscillatory problems
Lecture: 2 h, 4 credit points
Fri 11:30-13:00, 01.85, Z1
Prof. Schratz

0154600 Partial Differential Equations II
Lecture: 4 h, 8 credit points
Thu 8:00-9:30 05.20, 1C-04, Fri 8:00-9:30, 05.20 1C-04
Tutorial: 2 h
Wed 14:00-15:30 01.85, Z2
Prof. Lamm

0153500 Global differential geometry
Lecture: 4 h, 8 credit points
Wed 9:45-11:15, 01.85, Z2, Thu 9:45-11:15 01.85, Z2
Tutorial: 2 h
Fri 14:00-15:30 01.85,Z2
Prof. Tuschmann

0155200 Travelling Waves
Lecture: 3 h, 6 credit points
Tue 15:45-17:15 01.85, Z2
Tutorial: 1 h
Thu 11:30-13:00, 01.85, Z2
Prof. Rottmann-Matthes

0154100 Geometric Numerical Integration
Lecture: 2 h, 4 credit points
Thu 15:45-17:15 01.85, Z2
Tutorial: 1 h;
Wed 15:45-17:15 01.85, Z2
Prof. Jahnke

0155800 Numerical methods for Maxwell's equation
Lecture: 2 h, 4 credit points
Wed 8:00-9:30 01.85, Z2
Tutorial: 1 h;
Mon 14:00-15:30 01.85, Z2
Prof. Jahnke

0155700 Extreme value theory
Lecture: 3 h, 6 credit points
Mon 11:30-13:00 05.20, 1C-02
Tutorial: 1 h;
Thu 14:00-15:30 05.20, 1C-03
Prof. Vasen

0172000 Seminar: Graph Theory 6. Sem.
2 h, 3 credit points
Fri 14:00-15:30 5.20, 1C-02
Prof. Axenovich

0172910 Seminar: Spatial Stastics 6. Sem.
Blockseminar, date and time to be announced
Prof. Gneiting

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30			Num. Meth.	PDE II	PDE II
09:45-11:15			Gl. Diff. Geom.	Gl. Diff. Geom.	
11:30-13:00	Extr.Val.Theo.				Num. Ana.
14:00-15:30					
15:45-17:15		Trav. Waves		Geo. Num. Int.	

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: SUMMER SEMESTER 2014

0161300 Numerical Analysis of highly-oscillatory problems

Lecture: 2 h, 4 credit points
Fri 11:30-13:00, 01.85, Z1
Prof. Schratz

Contents

In this lecture we will firstly deepen our knowledge on splitting methods. Due to their computational advantage splitting methods are nowadays omnipresent in scientific computing. They pursue the intention to break down a complicated problem into a series of simpler subproblems. In the context of time integration a common idea is to split up the right-hand side and to decompose the given evolution equation into a sequence of subproblems, which in many situations can be solved far more efficiently or even exactly. The exact solution of the full-problem is then approximated by the composition of the flows associated to the simpler subproblems.

In this lecture we will discuss (amongst others):
Splitting methods for inhomogeneous evolution equations, their order reduction when ãnon-trivialÒ boundary conditions are present, as well as the derivation of correction methods in order to overcome the ãcurse of boundary conditions
High-order splitting methods for analytic semigroups
Splitting methods for the non-linear Schrödinger equation with polynomial nonlinearity
Modified energy of splitting schemes applied to linear Schrödinger equations with potential

Furthermore we will investigate efficient numerical time-integrators for highly-oscillatory problems such as
Averaging methods
Gautschi-type methods
Fourier expansion techniques

In the exercises we will deepen some theoretical results and carry out practical implementations.

Prerequisites:

One should be familiar with basic concepts of the numerical time integration of ODEs and PDEs and functional analysis. A basic knowledge of the theory of semigroups is helpful. Attendance of the lecture ãSplitting methodsÒ (winter term 2013/14) is helpful.

References: Will be given in the lecture.

0154600 Partial Differential Equations II

Lecture: 4 h, 8 credit points
Thu 8:00-9:30 05.20, 1C-04, Fri 8:00-9:30, 05.20 1C-04
Tutorial: 2 h
Wed 14:00-15:30 01.85, Z2
Prof. Lamm

Contents

In this lecture we continue our study of Elliptic Partial Differential Equations from the winter term and we discuss the following topics: Calderon-Zygmund theory, Schauder theory, DeGiorgi-Nash-Moser theory, Hardy- and BMO-spaces, Harmonic maps.

Prerequisites:

Students attending this course should have a basic knowledge of Sobolev spaces and harmonic functions.

References:

L.C. Evans, Partial Differential Equations, 2nd edition, 2010.
M. Giaquinta and L. Martinazzi, An introduction to the regularity theory for elliptic systems, harmonic maps and minimal graphs, 2nd edition, 2013.
D. Gilbarg and N. Trudinger, Elliptic Partial Differential Equations of second order, 1997.
Q. Han and F. Lin, Elliptic Partial Differential Equations, 2nd edition, 2011.
F. Hélein, Harmonic maps, conservation laws and moving frames, 2nd edition, 2002.

0153500 Global differential geometry

Lecture: 4 h, 8 credit points
Wed 9:45-11:15, 01.85, Z2, Thu 9:45-11:15 01.85, Z2
Tutorial: 2 h
Fri 14:00-15:30 01.85,Z2
Prof. Tuschmann

Contents

The course will cover various central themes of modern global differential geometry like
Existence and obstruction results for Riemannian metrics with special (curvature) conditions
Geometric classification and finiteness theorems
Geometry and topology of Riemannian manifolds with lower curvature bounds
Comparison geometry
Gromov-Hausdorff and Lipschitz convergence
and, if time will permit, some spin geometry and rudiments of Seiberg-Witten theory.

Prerequisites:

Thorough knowlegde of differentiable manifolds and first concepts of Riemannian Geometry like tensor bundles, connections, and curvature; basics of (Algebraic) Topology.

References:

R. Bott and L. Tu, Differential forms in algebraic topology. Graduate Texts in Mathematics 82, Springer-Verlag, New York-Berlin, 1982.

S. Gallot, D. Hulin and J. Lafontaine, Riemannian geometry. Third edition. Universitext, Springer-Verlag, Berlin, 2004.

M. Gromov, Metric structures for Riemannian and non-Riemannian spaces. Birkhäuser Boston, Inc., Boston MA, 1999.

H. B. Lawson and M.-L. Michelsohn, Spin geometry. Princeton Mathematical Series 38, Princeton University Press, Princeton NJ, 1989.

J. Milnor, Morse theory. Annals of Mathematics Studies 51, Princeton University Press, Princeton N.J., 1963.

T. Sakai, Riemannian geometry. Translations of Mathematical Monographs 149, American Mathematical Society, Providence RI, 1996.

C. Taubes, The geometry of the Seiberg-Witten invariants. Surveys in differential geometry, Vol. III (Cambridge, MA, 1996), 299 – 339, Int. Press, Boston MA, 1998.

0155200 Travelling Waves

Lecture: 3 h, 6 credit points
Tue 15:45-17:15 01.85, Z2
Tutorial: 1 h
Thu 11:30-13:00, 01.85, Z2
Prof. Rottmann-Matthes

Contents

In this lecture we consider travelling wave solutions of time-dependent partial differential equations posed on the real line. These are special solutions of the simple form $u(x,t) = \phi(x - ct)$, where ϕ is a fixed profile of the wave and c is its speed. The easiest example where such a solution arises is the linear transport equation

$$u_t = -cu_x, \qquad t \geq 0,\ x \in \mathbb{R}, \quad u(x,0) = \phi(x).$$

But these kind of solutions arise also in many other and, in particular, nonlinear problems.

We first consider several equations that give rise to travelling wave solutions. Then we will look at the large class of reaction-diffusion equations, which model many important phenomena, for example in biological systems. A main emphasis will be on the stability of travelling wave solutions. I.e. the time asymptotic behaviour of solutions with initial values close to the profile of a travelling wave. We will see that the stability is closely related to spectral properties of certain operators, obtained by linearization about the profile of the wave.

Therefore, we will then consider the spectral properties of such operators. A major difficulty is actually to locate the point spectrum and we introduce the Evans-function, which is an important tool for this. Because it is often not possible to analytically calculate the spectrum, we will also look at numerical methods that are well suited to approximate the spectrum.

In the final part of the lecture, we will also have a brief look at travelling waves in Hamiltonian systems.

Prerequisites:

The lecture assumes a fair knowledge of Analysis, Functional Analysis, Spectral theory, Semigroup theory, as well as a basic knowledge of Numerical linear algebra, Numerical analysis of ODEs.

References:

Throughout the lecture we will use
Sandstede, Stability of travelling waves. In: Handbook of dynamical systems, Vol. 2, 983–1055, North-Holland, Amsterdam, 2002;
Kapitula and Promislow, Spectral and dynamical stability of nonlinear waves, New York, NY, Springer, 2013,
as well as several original articles.

The following list gives some basic references and will be updated throughout the lecture:
Fife and McLeod, The approach of solutions of nonlinear diffusion equations to travelling front solutions, Arch. Ration. Mech. Anal., 65:335–361, 1977.
Grillakis, Shatah, and Strauss, Stability theory of solitary waves in the presence of symmetry. I., J. Funct. Anal., 74:160–197, 1987.
Henry, Geometric theory of semilinear parabolic equations, Volume 840 of Lecture Notes in Mathematics, Springer Berlin, 1981.
Knobel, An introduction to the mathematical theory of waves, Providence, AMS, 2000.
Sattinger, On the stability of waves of nonlinear parabolic systems, Advances in Math., 22(3):312–355, 1976.

0154100 Geometric Numerical Integration

Lecture: 2 h, 4 credit points
Thu 15:45-17:15 01.85, Z2
Tutorial: 1 h;
Wed 15:45-17:15 01.85, Z2
Prof. Jahnke

Contents

The numerical simulation of time-dependent processes in science and technology often leads to the problem to solve a system of ordinary differential equations (ODEs) with a suitable method. In many applications it can be shown that the exact solution or flow exhibits certain qualitative or "geometric" properties. For example, it is well-known that the exact flow of a Hamiltonian system is symplectic, and that the energy remains constant along the exact solution although the solution itself changes in time. When the solution or the flow is approximated by a numerical integrator, it is desirable to preserve these geometric properties (at least approximately), because reproducing the correct qualitative behavior is important in most applications. It turns out, however, that many numerical schemes destroy the structure of the solution, and that only selected methods respect the geometric properties of the dynamics. These methods are called geometric numerical integrators.

In this lecture we will investigate:
why certain methods are (or are not) geometric numerical integrators,
how to construct geometric numerical integrators,
which properties are conserved, and in which sense,
how structure conservation is related to the long-time error behavior of the method.

The course consists of a lecture (Thursday 15:45-17:15, Z 2) and exercise classes (Wednesday 14:00-15:30, computer pool in 01:93, every second week). Both the lecture and the exercise classes will be given in English.

Prerequisites:

The lecture will be suited for students in mathematics, physics and other sciences with a basic knowledge in ordinary differential equations and Runge-Kutta methods. In particular, students should be familiar with concepts such as, e.g., order, consistency, convergence, A-stability, and so on. The course "Numerische Methoden für Differentialgleichungen" provides a good basis. In the exercise class, students will be asked to write MATLAB programs which illustrate the theoretical results presented in the lecture. The exercises can be solved in pairs or alone, and with the assistance of the tutor.

References:

Hairer, Lubich, and Wanner, Geometric numerical integration. Structure-preserving algorithms for ordinary differential equations. 2nd edition, Springer, 2006.
Hairer, Lubich, and Wanner, Geometric numerical integration illustrated by the Störmer-Verlet method, Acta Numerica 12, 399-450, 2003.
Reich and Leimkuhler, Simulating Hamiltonian dynamics, Cambridge monographs on applied and computational mathematics 14. Cambridge University Press, 2004.

0155800 Numerical methods for Maxwell's equation

Lecture: 2 h, 4 credit points
Wed 8:00-9:30 01.85, Z2
Tutorial: 1 h;
Mon 14:00-15:30 01.85, Z2
Prof. Jahnke

Contents

The classical theory of electromagnetism is based on a system of coupled partial differential equations derived by James Clerk Maxwell about 150 years ago. These equations describe the interaction between the electric and magnetic fields, the electric and magnetic flux densities, and the electric and magnetic current densities. Solving Maxwell's equations numerically is challenging problem which appears in many different technical applications.

In this lecture numerical methods for the time-dependent Maxwell equations will be derived and analyzed. The first part will focus on finite-difference time-domain methods such as the famous Yee scheme and alternative approaches such as the ADI splitting method of Zheng, Chen, Zhang. The second part of the lecture willbe devoted to space discretization by finite elements and (if time permits) discontinuus Galerkin methods.

The course consists of a lecture (Wednesday 8:00-9:30, Z 2) and exercise classes (Monday 14:00-15:30, Z2, every second week). Both the lecture and the exercise classes will be given in English.

Prerequisites:

The lecture is intended for students in mathematics, physics and other sciences with basics in ordinary and partial differential equations and the corresponding numerical methods. Some background in functional analysis (Sobolev spaces) and operator semigroup theory is helpful, but not mandatory. Previous knowledge about Maxwell's equations is not expected because a short introduction to these equations and a proof of their well-posedness will be provided in the first chapter.

References:

Monk, Finite element methods for Maxwell's equations, Clarendon Press, Oxford, 2006.
Taflove and Hagness, Computational electrodynamics: the finite-difference time-domain method, 3rd edition, Artech House Boston, 2005.
Yee, Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media, IEEE Transactions on Antennas and Propagation, 14, 302 – 307, 1966.
Zheng, Chen, and Zhang, Toward the development of a three-dimensional unconditionally stable finite-difference time-domain method, IEEE Transactions on Microwave Theory and Techniques, 48, 1550–1558, 2000.

0155700 Extreme value theory

Lecture: 3 h, 6 credit points
Mon 11:30-13:00 05.20, 1C-02
Tutorial: 1 h;
Thu 14:00-15:30 05.20, 1C-03
Prof. Vasen

Contents

Understanding and managing risks caused by extreme events is one of the most demanding problems of our society. In the class we consider this topic from a statistical point of view and present some of the probabilistic and statistical theory, which was developed to model and quantify extreme events. By the very nature of an extreme event there will never be enough data to predict a future risk in the classical statistical sense. However, a rather clever probabilistic theory provides us with model classes relevant for the assessment of extreme events. Moreover, specific statistical methods allow for the prediction of rare events, even outside the range of previous observations.

Topics: Theorem of Fisher and Tippett, Generalized Extreme Value distribution (GED) and Generalized Pareto Distribution (GPD), maximum domain of attraction, Theorem of Pickands-Balkema-de Haan, estimation of risk measures, Hill-estimator, block-maxima method, POT method.

Prerequisites

The contents of the modul Wahrscheinlichkeitstheorie (Probability Theory).

References

Beirlant, Goegebeur, Segers, and Teugels, Statistics of Extremes. Wiley, 2004.
de Haan, and Ferreira, Extreme Value Theory: An Introduction, Springer, 2006.
Coles, An Introduction to Statistical Modeling of Extreme Values, Springer, 2001.
Embrechts, Klüppelberg, and Mikosch, Modelling Extremal Events for Insurance and Finance, Springer, 1997.´´ McNeil, Frey, and Embrechts, Quantitative Risk Management: Concepts, Techniques, and Tools, Princton University Press, 2005.