

# INTERNATIONAL PROGRAM (MASTER)

## CLASSES: SUMMER SEMESTER 2015

### **0150300 Combinatorics**

Lecture: 4 h, 8 credit points

Wed 9:45-11:15 20.40, NH, Thu 11:30-13:00, 20.40, NH

Tutorial: 2 h

Wed 14:00-15:30 20.40, HS 9

Dr. Ueckerdt

### **0157400 Algebraic Topology**

Lecture: 4 h, 8 credit points

Mon 15:45-17:15 20.30 SR 3.68, Wed 17:30-19:00 20.30 SR 2.59

Tutorial: 2 h

Fri 11:30-13:00 20.30 SR 2.59

Prof. Sauer

### **0157000 Maxwell Equations**

Lecture: 4 h, 8 credit points

Tue 11:30-13:00 20.30 SR 2.67, Wed 8:00-9:30 20.30 SR 2.67

Tutorial: 2 h

Wed 14:00-15:30 20.30 SR 2.67

Dr. Hettlich

### **0159610 Numerical methods in mathematical finance 2**

Lecture: 4 h, 8 credit points

Thu 8:00-9:30 20.30 SR 3.60, Fri 8:00-9:30 20.30 SR 3.60

Tutorial: 2 h

Mon 14:00-15:30, 01.93 Seminarraum K1

Prof. Jahnke

### **0160000 Probability Theory and Combinatorial Optimization**

Lecture: 4 h, 8 credit points

Mon 11:30-13:00 20.30 SR 2.58, Thu 9:45-11:15 20.30 SR 2.59

Tutorial: 2 h;

Wed 15:45-17:15 20.30 SR 2.59

Dr. Hug

### **0161300 Aspects of Numerical Time Integration**

Lecture: 2 h, 4 credit points

Thu 15:45-17:15, 20.30 SR 3.69

Tutorial: 2 h;

Tue 15:45-17:15 20.30 SR 3.69

JProf. Schratz

**0163700 Spectral Theory**

Lecture: 4 h, 8 credit points

Mon 9:45-11:15 20.30 SR 2.66, Wed 11:30-13:00 20.30 SR 2.66

Tutorial: 2 h;

Thu 15:45-17:15 20.30 SR 2.66

Prof. Schaubelt

**0178000 Forecasting: Theory and Practice II**

Lecture: 2 h, 4 credit points

Tue 14:00-15:30 20.30 SR 2.59

Tutorial: 2 h;

Tue 17:30-19:00 20.30 SR 2.59

Prof. Gneiting

**0167000 Numerical methods in computational electrodynamics**

Lecture: 2 h, 6 credit points

Wed 11:30-13:00, 20.30 SR 3.61

Tutorial: 2 h;

Tue 14:00-14:30 20.30 SR 3.61

Prof. Dörfler

**0161700 Projektorientiertes Softwarepraktikum**

4 h, 4 credit points

Tue 9:45-11:15, 01.93 Seminarraum K1, Fri 9.45-11:15, 01.93 Seminarraum K1

Dr. Thäter, Dr. Krause

**Time-table for lectures**

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30			Max.Equ.	Math.Fin.2	Math.Fin.2
09:45-11:15	Spec.Theo.	SoftwarePr.	Comb.	Prob.Theo.	Software-Pr.
11:30-13:00	Prof.Theo.	Max.Equ.	Spec.Theo. Comp.Elec.	Comb.	
14:00-15:30		Forec.2			
15:45-17:15	Al.Top.			Num.T.Integr.	
17:30-19:00			Al.Top.		

## GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

## CLASSES: SUMMER SEMESTER 2015

### 0150300 Combinatorics

Lecture: 4 h, 8 credit points

Wed 9:45-11:15 20.40, NH, Thu 11:30-13:00, 20.40, NH

Tutorial: 2 h

Wed 14:00-15:30 20.40, HS 9

Dr. Ueckerdt

#### Contents

This lecture is an introduction into combinatorics, a field concerned with the existence, enumeration, analysis and optimization of discrete structures. The students are taught various combinatorial techniques, which can be applied all over mathematics.

The specific topics include: counting and bijections, generating functions, partial orders, combinatorial designs and codes, Pólya theory.

#### Prerequisites:

Basic knowledge of linear algebra.

#### References:

R. A. Brualdi: Introductory Combinatorics.

J.H. van Lint, R. M. Wilson: A Course in Combinatorics.

### 0157400 Algebraic Topology

Lecture: 4 h, 8 credit points

Mon 15:45-17:15 20.30 SR 3.68, Wed 17:30-19:00 20.30 SR 2.59

Tutorial: 2 h

Fri 11:30-13:00 20.30 SR 2.59

Prof. Sauer

#### Contents

At the beginner's level, algebraic topology separates naturally into the two broad topics of homology and homotopy. The course covers the essentials of singular homology, including the axiomatic approach and the computational approach via cellular complexes. Basic properties of homotopy groups are discussed, but the focus is less on homotopy theory. Fundamental ideas of homological algebra will be an important part of the course. A highlight at the end will be the proof of Poincaré duality.

There will be oral exams of about 25 min after the end of the course.

**Prerequisites:**

Students are expected to be familiar with basic notions of set-theoretic topology like topological space, quotient topology, or manifold. The teaching concept is a mixture of an inquiry-based approach and a traditional lecture approach. This means that active participation is of utmost importance.

**References:**

G. E. Bredon: Topology and geometry, Graduate Texts in Mathematics, 139, Springer-Verlag, 1997, xiv+557.

T. Dieck: Algebraic topology, EMS Textbooks in Mathematics, EMS, Zürich, 2008, xii+567.

A. Hatcher: Algebraic topology, Cambridge University Press, 2002, xii+544. (available under <http://www.math.cornell.edu/hatcher/AT/ATpage.html>).

J. P. May: A concise course in algebraic topology, Chicago Lectures in Mathematics, University of Chicago Press, 1999, x+243.

**0157000 Maxwell Equations**

Lecture: 4 h, 8 credit points

Tue 11:30-13:00 20.30 SR 2.67, Wed 8:00-9:30 20.30 SR 2.67

Tutorial: 2 h

Wed 14:00-15:30 20.30 SR 2.67

Dr. Hettlich

**Contents**

A system of partial differential equations, the Maxwell equations, constitutes the basic concept of electro dynamics. The lecture will consider fundamental aspects of the mathematical theory of these differential equations. The potential theoretical approach as well as weak formulations of corresponding boundary value problems will be presented. Especially the theory of the scattering of time harmonic electromagnetic waves will be discussed.

**Prerequisites:**

Students of the master programs in Mathematics, Technomathematics, Physics or Electrical Engineering. Basic knowledge on partial differential equations and also on linear functional analysis are provided, but appropriate hints on literature will be given throughout the lecture.

**References:**

A. Kirsch and F. Hettlich: The Mathematical Theory of Time-Harmonic Maxwell's Equations, Springer, 2015.

W. McLean: Strongly Elliptic Systems and Boundary Integral Equations, Cambridge University Press, 2000.

P. Monk: Finite Element Methods for Maxwell's Equations, Oxford University Press, 2003.

## 0159610 Numerical methods in mathematical finance 2

Lecture: 4 h, 8 credit points

Thu 8:00-9:30 20.30 SR 3.60, Fri 8:00-9:30 20.30 SR 3.60

Tutorial: 2 h

Mon 14:00-15:30, 01.93 Seminarraum K1

Prof. Jahnke

### Contents

Based on the first part of this lecture given in the winter term, more models and methods for option pricing will be presented. The central theme is the construction and analysis of numerical methods which approximate the solution of the corresponding differential equations in a stable, accurate and efficient way.

The following topics will be discussed: Multi-level Monte-Carlo methods, Historical, implied and local volatility, Jump-diffusion models and integro-differential equations, Finite element methods for the Black-Scholes equation, Sparse grids for basket options and other high-dimensional problems.

The course consists of a lecture and problem class, which are both given in English. In the problem class the students are supposed to write short programs in order to test and apply the algorithms which will be presented in the lecture. Programming skills (preferably in MATLAB) are mandatory to solve these exercises.

### Prerequisites:

Participants should be familiar with part 1 of the lecture, see <http://www.math.kit.edu/ianm3/edu/numamathfin2014w/en>

### References:

Günther, Jüngel: Finanzderivate mit MATLAB. Mathematische Modellierung und numerische Simulation. Vieweg, 2010.

Seydel: Tools for computational finance, Springer 2009.

Brenner, Scott: The mathematical theory of finite element methods, Springer, Texts in Applied Mathematics 15, 2008.

Bungartz, Griebel: Sparse grids, Acta Numerica 13, 2004 (147-269). Cont, Tankov: Financial modelling with jump processes, Chapman and Hall, 2004.

Giles: Multilevel Monte Carlo Path Simulation, Operations Research 56/3, 2008 (608-617).

Giles: Improved multilevel Monte Carlo convergence using the Milstein scheme, in: Monte Carlo and quasi-Monte Carlo methods 2006. Springer 2008 (343-358).

Larsson, Thomée: Partial differential equations with numerical methods (paperback reprint), Springer, 2009.

Hilber, Reichmann, Schwab, Winter: Computational methods for quantitative finance. Finite element methods for derivative pricing. Springer, 2013. doi=10.1007/978-3-642-35401-4

## 0160000 Probability Theory and Combinatorial Optimization

Lecture: 4 h, 8 credit points

Mon 11:30-13:00 20.30 SR 2.58, Thu 9:45-11:15 20.30 SR 2.59

Tutorial: 2 h;

Wed 15:45-17:15 20.30 SR 2.59

Dr. Hug

### Contents

This course is devoted to the analysis of algorithms and combinatorial optimization problems in a probabilistic framework. A natural setting for the investigation of such problems is often provided by a (geometric) graph. For a given system (graph), the average or most likely behavior of an objective function of the system will be studied. In addition to asymptotic results, which describe a system as its size increases, quantitative laws for systems of fixed size will be described. Among the specific problems to be explored are the long-common-subsequence problem, packing problems, the Euclidean traveling salesman problem, minimal Euclidean matching, minimal Euclidean spanning tree.

For the analysis of problems of this type, several techniques and concepts have been developed and will be introduced and applied in this course. Some of these are concentration inequalities and concentration of measure, subadditivity and superadditivity, martingale methods, isoperimetry, entropy.

### Prerequisites:

Introduction to probability theory (on a measure theoretic foundation).

### References:

Boucheron, S., Lugosi, G., Massart, P. Concentration Inequalities, Oxford University Press, Oxford, 2013.

Dubhashi, D., Panconesi, A. Concentration of Measure for the Analysis of Randomized Algorithms, Cambridge University Press, Cambridge, 2009.

Ledoux, M. The Concentration of Measure Phenomenon. American Mathematical Society, vol. 89, 2001.

Steele, J.M. Probability Theory and Combinatorial Optimization. SIAM, 1997.

Yukich, J.E. Probability Theory of Classical Euclidean Optimization Problems. Lecture Notes in Mathematics, Vol. 1675, Springer, Berlin, 1998.

Yukich, J.E. Limit Theorems in Discrete Stochastic Geometry. In: Stochastic geometry, spatial statistics and random fields, 239-275, Lecture Notes in Mathematics, Vol. 2068, Springer, Heidelberg, 2013.

## 0161300 Aspects of Numerical Time Integration

Lecture: 2 h, 4 credit points

Thu 15:45-17:15, 20.30 SR 3.69

Tutorial: 2 h;

Tue 15:45-17:15 20.30 SR 3.69

JProf. Schratz

## Contents

In this lecture we will firstly deepen our knowledge on splitting methods. Due to their computational advantage splitting methods are nowadays omnipresent in scientific computing. They pursue the intention to break down a complicated problem into a series of simpler subproblems. In the context of time integration a common idea is to split up the right-hand side and to decompose the given evolution equation into a sequence of subproblems, which in many situations can be solved far more efficiently or even exactly. The exact solution of the full-problem is then approximated by the composition of the flows associated to the simpler subproblems.

In this lecture we will discuss (amongst others): Splitting methods for inhomogeneous evolution equations, their order reduction when non-trivial boundary conditions are present, as well as the derivation of correction methods in order to overcome the curse of boundary conditions; High-order splitting methods for analytic semigroups; Splitting methods for the non-linear Schrödinger equation with polynomial nonlinearity; Modified energy of splitting schemes applied to linear Schrödinger equations with potential; Furthermore we will investigate efficient numerical time-integrators for highly-oscillatory problems such as Averaging methods, Gautschi-type methods, Fourier expansion techniques.

In the exercises we will deepen some theoretical results and carry out practical implementations.

### Prerequisites:

One should be familiar with basic concepts of the numerical time integration of ODEs and PDEs and functional analysis. A basic knowledge of the theory of semigroups is helpful. Attendance of the lecture Splitting methods (winter term 2013/14) is helpful.

### References:

Will be given in the lecture.

## 0163700 Spectral Theory

Lecture: 4 h, 8 credit points

Mon 9:45-11:15 20.30 SR 2.66, Wed 11:30-13:00 20.30 SR 2.66

Tutorial: 2 h;

Thu 15:45-17:15 20.30 SR 2.66

Prof. Schaubelt

### Contents

The spectrum of a linear operator on a Banach space generalizes the concept of an eigenvalue of a matrix. In Banach spaces spectral theoretic methods play an equally important role as the eigenvalue theory in finite dimensions. These methods are used everywhere in analysis and its applications.

At the beginning we discuss the basic properties of the spectrum. In view of the applications on differential operators this is not only done for bounded operators, but also for a certain class of unbounded linear operators, the so-called closed operators. To treat differential operators on  $L^p$  spaces, we discuss the weak derivative in the  $L^p$  setting and Sobolev spaces. One can develop a detailed spectral theory for two main classes of operators: We first deal with

compact operators, where the spectrum is determined by the eigenvalues to a large extent. In this context we also prove the so-called Fredholm alternative, which has important applications to e.g. integral equations. Then we study (possibly only closed) selfadjoint operators on Hilbert spaces and establish the spectral theorem which is a far reaching extension of the diagonalisation of hermitian matrices. Finally, we treat the functional calculi for self adjoint, bounded and sectorial operators.

### Prerequisites

Functional analysis.

### References

- H.W. Alt: Lineare Funktionalanalysis, Springer.  
H. Brézis: Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer.  
J.B. Conway: A Course in Functional Analysis, Springer.  
N. Dunford, J.T. Schwartz: Linear Operators. Part I: General Theory, Wiley.  
T Kato: Perturbation Theory of Linear Operators, Springer.  
A.E. Taylor, D.C. Lay: Introduction to Functional Analysis, Wiley.  
D. Werner: Funktionalanalysis, Springer.

## 0178000 Forecasting: Theory and Practice II

Lecture: 2 h, 4 credit points  
Tue 14:00-15:30 20.30 SR 2.59  
Tutorial: 2 h;  
Tue 17:30-19:00 20.30 SR 2.59  
Prof. Gneiting

### Contents

A common desire of all humankind is to make predictions for the future. As the future is inherently uncertain, forecasts ought to be probabilistic, i.e., they ought to take the form of probability distributions over future quantities or events. In this course, which comprises the second part of a series, we will continue to study the probabilistic and statistical foundations of the science of forecasting.

The goal in probabilistic forecasting is to maximize the sharpness of the predictive distributions subject to calibration, based on the information set at hand. Proper scoring rules such as the logarithmic score and the continuous ranked probability score serve to assess calibration and sharpness simultaneously, and relate to information theory and convex analysis. As a special case, consistent scoring functions provide decision-theoretically coherent tools for evaluating point forecasts. Furthermore, we will study methodological links to parametric and nonparametric distributional regression techniques, where the goal is to model and estimate conditional distribution functions, and we will discuss the aggregation of probabilistic forecasts. Throughout, concepts and methodologies will be illustrated in data examples. Furthermore, we will entertain a probabilistic forecast competition.



## Prerequisites

A firm understanding of the contents of part I of the sequence (Forecasting: Theory and Practice I) is essential.

## References

Non-technical overviews of the topics covered are available in an editorial (Gneiting 2008) and a recent review paper (Gneiting and Katzfuss 2014). Technical references include the papers by Gneiting and Raftery (2007), Gneiting (2011) and Gneiting and Ranjan (2013).

Gneiting, T. (2008). Editorial: Probabilistic forecasting. *Journal of the Royal Statistical Society Series A: Statistics in Society*, **171**, 319–321.

Gneiting, T. (2011). Making and evaluating point forecasts. *Journal of the American Statistical Association*, **106**, 746–762.

Gneiting, T. and Katzfuss, M. (2014). Probabilistic forecasting. *Annual Review of Statistics and its Application*, **1**, 125–151.

Gneiting, T. and Raftery, A. E. (2007). Strictly proper scoring rules, prediction, and estimation. *Journal of the American Statistical Association*, **102**, 359–378.

Gneiting, T. and Ranjan, R. (2013). Combining predictive distributions. *Electronic Journal of Statistics*, **7**, 1747–1782.

## 0167000 Numerical methods in computational electrodynamics

Lecture: 2 h, 6 credit points  
Wed 11:30-13:00, 20.30 SR 3.61  
Tutorial: 2 h;  
Tue 14:00-14:30 20.30 SR 3.61  
Prof. Dörfler

## Contents

We focus on theory and numerics of Maxwell equations for the approximation of photonic bandstructures or waveguides. The Maxwell system, aspects of modeling. Boundary and interface conditions. Analytical tools. The curl curl source problem. The curl curl eigenvalue problem. Finite element methods for the Maxwell equations. Bandstructure computations, simulation and optimisation of waveguides. Oral examination (about 25 min).

## Prerequisites

Basic lectures in Partial Differential Equations and Numerical Methods of Partial Differential Equations.

## References

P. Monk: Finite Element Methods for Maxwell's Equations.

J. S. Hesthaven, T. Warburton: Nodal discontinuous Galerkin methods.

R. Hiptmair: Finite elements in computational electromagnetism. *Acta Numerica*, 11:237-339, 2002.

## 0161700 Projektorientiertes Softwarepraktikum

4 h, 4 credit points

Tue 9:45-11:15, 01.93 Seminarraum K1, Fri 9.45-11:15, 01.93 Seminarraum K1

Dr. Thäter, Dr. Krause

### Contents

In the tutorial, you will work on two projects dealing with Computational Fluid Dynamics and Lattice Boltzmann methods. We will solve these problems using the OpenSource framework OpenLB (<http://www.openlb.net>). You can work alone or in pairs. In order to obtain the ECTS, you must successfully complete the two projects as detailed below.

Requirements: Mandatory presence at the dates listed below; Successful completion of the two projects.

For each project, you will have to: Give a short presentation (10/20 min); Hand in a report describing your solutions to the exercises, including: Mathematical problem formulations; Reasoning supported by analytical derivations; Plots and tables of computed solutions and other values.

Hand in your source code, extensively commented. The report should be written with  $\text{\LaTeX}$ . The report, code and presentation can be in English or German.

### Sessions with mandatory presence:

Tue	14.04.15	Introduction to tutorial and Lattice Boltzmann Methods.
Fri	17.04.15	Introduction to OpenLB and start project 1.
Fri	01.05.15	Public holiday - <b>No tutorial</b> .
Tue	19.05.15	Presentation and hand-in report + source code project 1.
Fri	22.05.15	Start project 2.
Tue	10.07. & 14.07.15	Presentation and hand-in report + source code project 2.

### Prerequisites

Solid knowledge of C++. Interest in Lattice Boltzmann Methods.

### References

On Boltzmann's equation und LBM:

D. Hänel: Molekulare Gasdynamik. Springer, 2004.

D. Wolf-Gladrow: Lattice-Gas Cellular Automata and Lattice Boltzmann Models: An Introduction. Springer, Berlin Heidelberg, 2010.

A.A. Mohamad: Lattice Boltzmann Method: Fundamentals and Engineering Applications with Computer Codes. Springer, 2011.

H. Babovsky: Die Boltzmann-Gleichung. Teubner Verlag, 2002.

C. Cercignani: The Boltzmann Equation and its Applications. Springer-Verlag, New York, 1988.