INTERNATIONAL PROGRAM (MASTER)

CLASSES: SUMMER SEMESTER 2012

0160800 Numerical methods for hyperbolic equations
Lecture: 2 h, 4 credit points
Wed 11:30-13:00 5.20 1C-04
Prof. Willy Dörfler

0153500 Global Differential Geometry
Lecture: 4 h, 8 credit points
Tue 14:00-15:40 01.85 Z1, Thu 11:30-13:00 01.85 Z1
Tutorial: 2 h
Fri 14:00-15:30 01.85 Z1
Prof. Wilderich Tuschmann

0157500 Boundary and Eigenvalue Problems
Lecture: 4 h, 8 credit points
Mon 11:30-13:00 5.20 1C-04, Tue 9:45-11:15 5.20 1C-04
Tutorial: 2 h
Wed 14:00-15:30 01.85 Z1
Prof. Michael Plum

0156600 Computer Assisted Proofs for Partial Differential Equations
Lecture: 4 h, 8 credit points
Mon 14:00-15:30 5.20 1C-04, Fri 9:45-11:15 5.20 1C-04
Ass. Prof. Kaori Nagatou

0164100 Fourier Analysis
Lecture: 4 h, 8 credit points
Wed 9:45-11:15 5.20 1C-03, Thu 14:00-15:30 30.32 RPH R. 045
Prof. Maria Girardi

0156900 Integral Equations
Lecture: 4 h, 8 credit points
Mon 9:45-11:15 01.85 Z1, Thu 9:45-11:15 01.85 Z1
Tutorial: 2 h
Mon 15:45-17:15 01.85 Z1
Dr. Frank Hettlich

0154000 Wave Equations
Lecture: 4 h, 8 credit points
Tue 11:30-13:00 5.20 1C-04, Thu 8:00-9:30 5.20 1C-04
Prof. Tobias Lamm

0159900 Markov Decision Processes
Lecture: 2 h, 4 credit points
Mon 8:00-9:30 01.85 Z1
Tutorial: 2 h
Fri 11:30-13:00 01.85 Z1
Prof. Nicole Bäuerle

0154100 Geometric Numerical Integration
Lecture: 2 h, 4 credit points
Tue 8:00-9:30 5.20 1C-03
Tutorial: 2 h
Thu 15:45-17:15 5.20 1C-04
Dr. David Cohen

054300 Iterative Methods for Sparse Linear Systems
Lecture: 2 h, 4 credit points
Tue 15:45-17:15 01.85 Z2
Dr. Jan Meyer

0175400 Seminar: Engineering Mathematics and Computing
Thu 14:00-15:30
Prof. Vincent Heuveline, Dr. Gudrin Thäter

0172100 Seminar: Graph Theory
Wed 9:45-11:15 5.20 1C-01
Prof. Maria Axenovich

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30		Geo.Num.Int.	Markov	Wave Equ.	
09:45-11:15	Int.Equ.	BEP	Four.Ana.	Int.Equ.	C.A. PDE
11:30-13:00	BEP	Wave Equ.	Num. Meth.	Glob. Diff. G.	
14:00-15:30	C.A. PDE	Glob. Diff. G.		Four.Ana.	
15:45-17:15		It.Meth.			

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: SUMMER SEMESTER 2012

0160800 Numerical methods for hyperbolic equation

Lecture: 2 h, 4 credit points
Wed 11:30-13:00 5.20 1C-04
Prof. Willy Dörfler

Contents

We present basic theory for equations in conservation form and the fundamental principle to derive numerical methods. As an application we focus on Burger's equation and Maxwell equations.

- Derivation of equations in conservation form
- Shocks, Rarefaction waves, weak solutions
- Aspects of existence and regularity theory
- Discretization of conservation laws
- Application to Maxwell equations

Prerequisites: For students above 6. semester.

References

J. S. Hesthaven, T. Warburton: Nodal discontinuous Galerkin methods
D. Kröner: Numerical Schemes for Conservation Laws
R. Leveque: Numerical Methods for Conservation Laws

0153500 Global Differential Geometry

Lecture: 4 h, 8 credit points
Tue 14:00-15:40 01.85 Z1, Thu 11:30-13:00 01.85 Z1
Tutorial: 2 h
Fri 14:00-15:30 01.85 Z1
Prof. Wilderich Tuschmann

Contents

The course will cover various central themes of modern global differential geometry like de Rham and Hogde theory, geometric finiteness theorems, geometry and topology of Riemannian manifolds with lower curvature bounds, comparison geometry, Alexandrov spaces, Gromov-Hausdorff covergence, and, if time will permit, spin geometry and rudiments of Seiberg-Witten theory.

Prerequisites

Thorough knowlegde of differentiable manifolds and first concepts of Riemannian Geometry like bundles, connections, and curvature; basics of Algebraic Topology.

References

R. Bott, L. Tu: Differential forms in algebraic topology. Graduate Texts in Mathematics 82, Springer-Verlag, New York-Berlin, 1982.
S. Gallot, D. Hulin und J. Lafontaine: Riemannian geometry. Third edition. Universitext. Springer-Verlag, Berlin, 2004.
M. Gromov: Metric structures for Riemannian and non-Riemannian spaces. Birkhäuser Boston, Inc., Boston, MA, 1999.´´ H.B. Lawson und M.-L. Michelsohn: Spin geometry. Princeton Mathematical Series 38, Princeton University Press, Princeton, NJ, 1989.
J. Milnor: Morse theory. Annals of Mathematics Studies 51, Princeton University Press, Princeton, N.J. 1963.
T. Sakai: Riemannian geometry. Translations of Mathematical Monographs 149, American Mathematical Society, Providence, RI, 1996.
C. Taubes: The geometry of the Seiberg-Witten invariants. Surveys in differential geometry, Vol. III (Cambridge, MA, 1996), 299–339, Int. Press, Boston, MA, 1998.

0157500 Boundary and Eigenvalue Problems

Lecture: 4 h, 8 credit points
Mon 11:30-13:00 5.20 1C-04, Tue 9:45-11:15 5.20 1C-04
Tutorial: 2 h
Wed 14:00-15:30 01.85 Z1
Prof. Michael Plum

Contents

A boundary value problem consists of an elliptic (or ordinary) differential equation posed on some domain, together with additional conditions required on the boundary of the domain, e.g. prescribed values for the unknown function. Typical origins of boundary value problems are steady-state (i.e. time-independent) situations in physics and engineering. An eigenvalue problem for a differential equation is a linear and homogeneous boundary value problem depending (typically linearly) on an additional parameter, and one is inter-ested in values of this parameter such that the boundary value problem has nontrivial solu-tions. Eigenvalue problems arise e.g. after separation of variables in time-dependent prob-lems (thus describing many vibrational situations, including quantum mechanics). The lectures will be accompanied by exercise lessons. Attendance of these exercises is strongly recommended to all participants.

Prerequisites

The lecture course addresses students in their fourth semester (second year) or higher, with substantial knowledge in analysis and linear algebra.

References

A. Friedman: Partial Differential Equations (general elliptic PDE of order 2m, but smooth data only)
D. Gilbarg, N. Trudinger: Elliptic Partial Differential Equations of Second Order (elliptic PDE of second order, mainly Dirichlet b.c.)
L. C. Evans: Partial Differential Equations
R. A. Adams: Sobolev Spaces (no PDE's, but excellent and general introduction into Sobolev spaces, an essential tool in PDE theory)

0156600 Computer Assisted Proofs for Partial Differential Equations

Lecture: 4 h, 8 credit points
Mon 14:00-15:30 5.20 1C-04, Fri 9:45-11:15 5.20 1C-04
Ass. Prof. Kaori Nagatou

Contents

Mathematical models in form of differential equations play an essential role in science and engineering, and investigating their solutions is of high importance in various respects, including the basic concepts of industrial technologies. Many analytical methods have been (and are being) developed over the centuries which give answers to questions concerning existence, multiplicity, and qualitative and quantitative properties of solutions to various classes of differential equation problems, such as variational methods, index and degree theory, monotonicity methods, fixed-point methods, semi-group methods, and more. Nevertheless, many important problems from theory and practice are still lacking satisfactory analytical results. On the other hand, numerical methods for computing approximate solutions to differential equation problems have enjoyed a huge development over the last decades, ranging from finite differences and projective spectral methods to finite elements and finite volumes, often in combination with multigrid schemes and specialized numerical linear algebra.
So many differential equation problems allow very ßtable"numerical computations of approximate solutions, but are lacking results which are reliable in a strict mathematical sense. The field of "computer-assisted proofsör "verified computations", which has attracted much attention in recent years, exploits the knowledge of "good"numerical approximate solutions to produce, by additional analytical arguments, strict mathematical results e.g. in form of existence statements together with rigorous error bounds. This field is by nature located near the borderline between analysis and numerical mathematics, which makes it very interesting and, since strengths of both sides are combined, also powerful.
The lecture course aims at the introduction of basic principles of computer-assisted proofs

for differential equations based on a functional analytical setting, and also at rigorous computational techniques by use of interval arithmetic.
A written or oral examination (depending on the number of attendees) will be held in the end of the semester.

Prerequisites

The lecture course addresses students in their sixth semester or higher, with basic knowledge in functional analysis and partial differential equations. It is suitable for students of mathematics, and for students of other subjects who have strong mathematical interests.

References

More papers will be introduced during the lecture course.
Robert A. Adams, John J. F. Fournier, SOBOLEV SPACES, second edition, Academic Press, 2003.
Götz Alefeld and Jürgen Herzberger, INTRODUCTION TO INTERVAL COMPUTATIONS, Academic Press, 1983.
Michael Plum, Computer-assisted Proofs for Semilinear Elliptic Boundary Value Problems. Japan Journal for Industrial and Applied Mathematics, Vol. 26, No. 2-3 (2009).

0164100 Fourier Analysis

Lecture: 4 h, 8 credit points
Wed 9:45-11:15 5.20 1C-03, Thu 14:00-15:30 30.32 RPH R. 045
Prof. Maria Girardi

Contents

Fourier Series, Convolution and Approximate Identity, Fourier Transform, Schwartz Class and Distributions, Interpolation Theorems, Fourier Multiplier Operators

Prerequisites

Analysis I, II, III.

References

Elias M. Stein and Rami Shakarchi, Fourier Analysis. Princeton University Press, 2003.
Loukas Grafakos, Classical and Modern Fourier Analysis, Prentice Hall, 2004.
Gerald Folland, Real Analysis: Modern Techniques and Their Applications, John Wiley & Sons, 2. Ed. (1999)

0156900 Integral Equations

Lecture: 4 h, 8 credit points

Mon 9:45-11:15 01.85 Z1, Thu 9:45-11:15 01.85 Z1
Tutorial: 2 h
Mon 15:45-17:15 01.85 Z1
Dr. Frank Hettlich

Contents

Besides the differential equations also integral equations are important mathematical concepts in physical, technical or medical applications. The formulation of boundary value problems by integral equations are often of theoretical interest (existence and uniqueness of solutions) as well as a basis of efficient numerical solution schemes. The lecture will give an introduction to a functional analytic approach to linear integral equations. We will discuss fundamental typs like Volterra equations, Fredholm equations and convolution equations. Additionally there will be problem sheets and a problem session, where we work on the problems. Afterwards also suggestions of solutions to the problems will be offered.

Prerequisites

It is a class for students of any mathematics programm (6.-10. semester). A prerequesite to understand the material of this class are the basic lectures of the Bachelor in Mathematics. Necessary results from functional analysis will be discussed in the lecture. In fact, this class can be seen as an expansion and application of the concepts and methods of functional analysis.

References

Presumably there will be a scriptum by the end of the semester (in German). Additionally, the lecture is based in parts on the following textbooks.
R. Kress, Linear Integral Equations, Springer, 1989.
H. Hochstadt, Integral Equations, Wiley, 1973.

0154000 Wave Equations

Lecture: 4 h, 8 credit points
Tue 11:30-13:00 5.20 1C-04, Thu 8:00-9:30 5.20 1C-04
Prof. Tobias Lamm

Contents

The wave equation is a second order partial differential equation which arises naturally in physics, for example in fields such as accoustics, electromagnetics and fluid dynamics.
The goal of this course is to develop the analytic foundations which are necessary in order to understand wave equations. At the beginning of the course we look at classical solutions of the standard wave equation. Then we move on to nonlinear equations and a typical result we are going to show is the local existence of solutions with small initial data. We also study in some detail the equation

$$u_{tt} - \Delta u + u^5 = 0$$

in $\mathbb{R}^3 \times (0, \infty)$. Time permitting we will then move on to Geometric Wave Equations, in particular Wave maps.

Prerequisites

Students attending this course should have a basic knowledge of Partial Differential Equations and Sobolev spaces. For the last part on Geometric Wave Equations a basic knowledge of Riemannian Geometry will be helpful.

References

L.C. Evans, Partial Differential Equations, 2nd edition, 2010.
J. Shatah and M. Struwe, Geometric Wave Equations, 2000.
C. Sogge, Lectures on Non-Linear Wave Equations, 2nd edition, 2008.
T. Tao, Nonlinear Dispersive Equations: Local and Global Analysis, 2006.

0159900 Markov Decision Processes

Lecture: 2 h, 4 credit points
Mon 8:00-9:30 01.85 Z1
Tutorial: 2 h
Fri 11:30-13:00 01.85 Z1
Prof. Nicole Bäuerle

Contents

Suppose a system is given which can be controlled by sequential decisions. The state transitions are random and we assume that the system state process is Markovian which means that previous states have no influence on future states. Given the current state of the system (which could be for example the wealth of an investor) the controller or decision maker has to choose an admissible action (for example a possible investment). Once an action is chosen there is a random system transition according to a stochastic law (for example a change in the asset value) which leads to a new state. Markov Decision Processes theory deals with controlling the process in such a way that the expected discounted reward of the system is maximized. We will consider problems with finite and infinite horizon, stopping problems and problems with partial observation. Applications in the area of portfolio optimization, queueing and operations research are considered.

Prerequisites

Excellent knowledge of probability theory. Basic knowledge in Markov chains is helpful.

References

Bäuerle, N. and Rieder, U. (2011): Markov Decision Processes with applications to finance. Springer-Verlag.
Bertsekas, D. P. (2001) Dynamic programming and optimal control. Vol. II. Athena Scientific.
Bertsekas, D. P. (2005) Dynamic programming and optimal control. Vol. I. Athena Scientific.
Hernández-Lerma, O. and Lasserre, J. B. (1996) Discrete-time Markov control processes. Springer-Verlag.
Hernández-Lerma, O. and Lasserre, J. B. (1999) Further topics on discrete-time Markov control processes. Springer-Verlag.
Puterman, M. L. (1994) Markov decision processes: discrete stochastic dynamic programming. John Wiley & Sons.
Ross, S. (1983) Introduction to stochastic dynamic programming. Academic Press.
Schäl, M. (1990) Markoffsche Entscheidungsprozesse. B. G. Teubner.

0154100 Geometric Numerical Integration

Lecture: 2 h, 4 credit points
Tue 8:00-9:30 5.20 1C-03
Tutorial: 2 h
Thu 15:45-17:15 5.20 1C-04
Dr. David Cohen

Contents

Ordinary differential equations often appear in the dynamical description of systems in physics, chemistry, biology, etc. Many differential equations exhibit geometric properties that are preserved by the dynamics. Recently, there has been a trend towards the construction of geometric numerical integrators. Such methods are of particular interest in the simulation of mechanical systems, with the preservation of invariants as the energy, momentum or symplectic form is important, especially in long-term simulations (Figure 1).

Topics: Numerical methods for ordinary differential equations; Hamiltonian problems; Structure-preserving numerical integrators; Highly oscillatory differential equations.

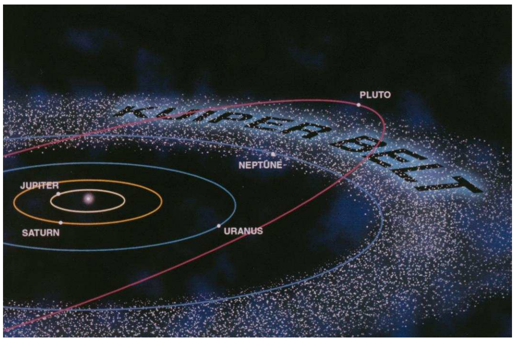

Abbildung 1: The outer solar system (www.cnes.fr)

Prerequisites

Master students, advanced Diploma students, members of the Research Training Group. Students from physics and other sciences with a basic knowledge in ordinary differential equations are welcome.

References

E. Hairer, C. Lubich, G. Wanner: Geometric Numerical Integration
B. Leimkuhler, S. Reich: Simulating Hamiltonian Dynamics
E. Hairer, C. Lubich, G. Wanner: Geometric Numerical Integration Illustrated by the Störmer-Verlet Method, 2003, www.unige.ch/ hairer/preprints.html
E. Hairer: Geometric Numerical Integration, script: www.unige.ch/ hairer/polycop.html

054300 Iterative Methods for Sparse Linear Systems

Lecture: 2 h, 4 credit points
Tue 15:45-17:15 01.85 Z2
Dr. Jan Meyer

Contents

Numerical algorithms for finding an approximate solution to some real life problems often yield linear systems having only a few non-zero coefficients in each equation. Such systems are called sparse. They occur in particular for problems having 2D or 3D geometry and whose solution can be described by partial differential equations. The variables describing an approximation of the solution usually depend only on variables that are "closeïn the underlying geometry, but not on any variables that are farther away. As a consequence, only very few variables actually occur in any linear equation. Similarly, any problem that can be described using graphs (e.g. networks, circuits) often results in sparse linear systems. More specifically, sparse linear systems occur in electrical circuit and semiconductor device simulation, fluid dynamics problems, chemical process simulation, macroeconomic modeling and many other applications.
As the coefficient matrix of these systems is large and sparse, typically having no more than 2-100 non-zero elements per row on average and generally having a dimension of at least

100,000 and often much more, simply using Gaussian elimination to solve the system is not feasible. Gaussian elimination does not exploit the large number of zeroes, so that computation time is generally unacceptably high. More importantly, computers often do not have the memory needed to store an entire matrix of this size. Instead, it is usually only possible to store the non-zero elements. As a consequence, we need algorithms for solving a linear system which only make use of these non-zero elements. Iterative methods, requiring only matrix-vector-multiplications with the coefficient matrix, satisfy this condition because all zero elements can be ignored for calculating matrix-vector-products.
In this course, we will cover different iterative methods for solving sparse linear systems. After considering some simple approaches, such as the minimal residual or steepest descent methods, we will discuss more sophisticated projection methods with a special focus on Krylov subspace methods such as GMRES and CG. Special emphasis will be placed on the advantages and disadvantages of each method and on convergence.
The continuation of this course will be offered in the following semester. The emphasis of that course will be on further Krylov space methods and on improving convergence by preconditioning.

Prerequisites

The course requires a good working background in linear algebra. Previous experience with numerical mathematics is helpful but not essential.

References

Axelson, Owe: Iterative Solution Methods.
Meurant, Gerard: Computer Solution Methods for Large Linear Systems.
Saad, Yousef: Iterative Methods for Sparse Linear Systems, 2nd Edition.
van der Vorst, Henk: Iterative Krylov Methods for Large Linear Systems.