INTERNATIONAL PROGRAM (MASTER)

CLASSES: WINTER SEMESTER 2012/2013

0109500 Mathematical Modelling and Simulation
Lecture: 2 h, 4 credit points
Wed 9:45-11:15 01.85 Z2 (17.10.-7.2.)
Prof. Gudrun Thäter

0109700 Parallel computing and numerics
Lecture: 4 h, 8 credit points
Feb 11-15, 2013 - place and time to be announced
Dr. Jan-Philipp Weiß

0104800 Functional Analysis
Lecture: 4 h, 8 credit points
Tue 9:45-11:15 10.23 Nusselt (16.10.-5.2.), Thu 11:30-13:00 10.11 Hertz (18.10.-7.2.)
Tutorial: 2 h
Fri 14:00-15:30 20.40 Eiermann (19.10.-8.2.)
Prof. Dirk Hundertmark

0107300 Introduction into Maxwell's Equations
Lecture: 4 h, 8 credit points
Mon 11:30-13:00 05.20 1C-03 (15.10.-4.2.), Wed 11:30-13:00 05.20 1C-03 (17.10.-6.2.)
Tutorial: 2 h
Fri 11:30-13:00 01.85 Z2 (19.10.-8.2.)
Prof. Andreas Kirsch

0100300 Selected Topics in Geometric Group Theory: The mapping class group
Lecture: 4 h, 8 credit points
Tue 11:30-13:00 01.85 Z1, Wed 8:00-9:30 05.20 1C-03
Tutorial: 2 h;
Wed 14:00-15:30 01.85 Z1 (17.10.-6.2.)
Prof. Gabriela Weitze-Schmithüsen

0105800 Brownian Motion
Lecture: 2 h, 4 credit points
Thu 9:45-11:15 5.20 1C-04 (18.10.-7.2.)
Tutorial: 1 h;
Wed 15:45-17:15 01.85 Z2 (17.10.-6.2.)
Prof. Vicky Fasen

0108000 Numerical methods in mathematical finance
Lecture: 4 h, 8 credit points
Mon 14:00-15:30 5.20 1C-04 (15.10.-4.2.), Thu 8:00-9:30 05.20 1C-04 (18.10.-7.2.)
Tutorial: 2 h;
Fri 11:30-11:15 05.20 1C-04 (19.10.-8.2.)
Prof. Tobias Jahnke

0109800 Iterative Methods for Sparse Linear Systems
Lecture: 2 h, 4 credit points
Tue 15:45-17:15 01.93 K2 (16.10.-5.2.)
Dr. Jan Mayer

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30			Geom.Gr.Th.	Math. Fin.	
09:45-11:15		Func. Ana.	Math.Mod.Sim.	Br. Motion	
11:30-13:00	Maxwell	Geom.Gr.Th.	Maxwell	Func. Ana.	
14:00-15:30	Math. Fin.				
15:45-17:15		It. Meth.			

not in timetable: block lecture: Par. Comp., Weiß

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: WINTER SEMESTER 2012/2013

0109500 Mathematical Modelling and Simulation

Lecture: 2 h, 4 credit points
Wed 9:45-11:15 01.85 Z2 (17.10.-7.2.)
Prof. Gudrun Thäter

Contents

Mathematics as way of thinking (via modeling) and as technique (i.e. providing tools) meets problems arising in everyday life. The problems themselves are easy to understand and the lecture will not rely on too much previous knowledge. Basic understanding of probability and Ordinary Differential equations will do. But you should bring along some enthusiasm to use computers. Themes will comprise
1) difference equations
2) population models
3) traffic modeling
4) growth modeling
5) game theory
6) chaos
7) problems in mechanics and fluid dynamics
Learning outcomes: 1) Broad horizon of modelling tools, 2) (Un)stability and (un)reliability of models, 3) Adequate accuracy and verification

References

lecture notes (in english)

0109700 Parallel computing and numerics

Lecture: 4 h, 8 credit points
Feb 11-15, 2013 - place and time to be announced
Dr. Jan-Philipp Weiß

Contents

Modern supercomputers and the development of adapted parallel algorithms facilitate solution of problems within a few hours that would be unsolvable on desktop computers or would take weeks or months. Thus, simulation of complex phenomenons in natural and engineering sciences and corresponding parameter studies are enabled. This lecture is treating basics of efficient utilization of parallel computers. Special focus is set to mathematical modelling and domain decomposition methods for the solution of partial differential equations.

Hardware architectures and parallel programming paradigms and environments for systems with shared and distributed memory are introduced. For problem statements in numerical mathematics we show parallel data structures (e.g for tridiagonal and sparse matrizes) and parallel algorithms. In particular, we detail parallel solvers for linear systems of equations. Furthermore, we discuss methods for assessment of performance and recent developments in multi-core technologies (e.g. GPUs, FPGAs, MIC). This lecture is providing an introduction to parallel computing and programming approaches in terms of MPI, OpenMP and CUDA. In the accompanying practical class it is dealt with different problems on parallel computers by means of MPI, OpenMP and CUDA.
After successfully completing this course, students will be able to:

- Implement linear algebra routines on parallel computers by means of MPI, OpenMP and CUDA
- Solve problems related to partial differential equations on parallel computers
- Handle parallel computing platforms, paradigms and libraries
- Have good knowledge in parallel solution techniques for linear systems of equations
- Report on theoretical and practical experience on parallel computers
- Conduct performance analysis and scalability assessment

Prerequisites

Necessary prerequisites are basic knowledge in programming languages C or C++. The required knowledge in numerical mathematics and treatment of partial differential equations can be acquired within the course.
The following courses are required as prerequisites: Programming in C/C++ (optional: Java)
The following courses are recommended as prerequisites: Numerical Mathematics I,II, Numerics of Partial Differential Equations
Related courses: Introduction to Scientific Computing, Software-Lab Fluid Mechanics

References

T. Mattson et al: Patterns for Parallel Programming. Addisson-Wesley, 2004
C. Lin: Principles of Parallel Programming. Pearson, 2009
C. Breshears: The Art of Concurrency. Reilly, 2009
J. Schüle: Paralleles Rechnen. Oldenbourg 2010
G. Alefeld et. al.: Parallele numerische Verfahren. Springer, 2002
W. Huber: Paralleles Rechnen. Oldenbourg, 1997
H. Schwandt: Parallele Numerik. Teubner, 2003
T. Rauber et al.: Parallele Programmierung, Springer, 2007

0104800 Functional Analysis

Lecture: 4 h, 8 credit points
Tue 9:45-11:15 10.23 Nusselt (16.10.-5.2.), Thu 11:30-13:00 10.11 Hertz (18.10.-7.2.)
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Prof. Dirk Hundertmark

Contents

The lectures on Functional Analysis will cover the fundamental concepts of metric spaces, Banach spaces, the Hahn-Banach separation theorem, open mapping theorem, uniform boundedness principle, the closed range theorem, duality and compactness. Our approach will be less focused on discussing the most abstract concept in detail, but we will discuss the extremely powerful and versatile tools Functional Analysis gives for Applied Mathematics, in particular, a modern approach to Partial Differential Equations.

References

The book we will mainly use is 'Functional Analysis, Sobolev Spaces and Partial Differential Equations' by Haim Brezis. See the vivid discussion of this book on amazon:
http://www.amazon.de/Functional-Analysis-Differential-Equations-Universitext/dp/0387709134/ref=sr_1_1?ie=UTF8&qid=1349869660&sr=8-1

0107300 Introduction into Maxwell's Equations

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Prof. Andreas Kirsch

Contents

Electromagnetic wave propagation is modeled by a system of two partial differential equations for the electric and magnetic fields E and H, the Maxwell system. This will be the starting point of our lecture. We will consider time-harmonic fields (i.e. periodic in time) solely and study two model problems in detail: First, we investigate a scattering problem and treat this special boundarzy value problem in an unbounded domain by a boundary integral equation method. Second, we study a cavity problem; that is, a boundary value problems in a bounded domain, and treat this by a variational method. For this part we will make use of Sobolev spaces, which will be introduced at the beginning of this section.

Prerequisites

Vordiplom in Mathematics, Physics or in Engineering
Basic knowledge of functional analysis is needed, in particular the notions of normed spaces, Hilbert spaces including their most important examples ($C(\overline{D})$, $L^2(D)$), linear and bounded or compact operators, the representations theorem of Riesz in Hilbert spaces. Further facts on functional analysis will be derived during the course.

References

D. Colton, R. Kress: Inverse Acoustic and Electromagnetic Scattering Theory. 2nd edition. Springer Verlag, 1998.
P. Monk: Finite Element Methods for Maxwell's Equations. Oxford University Press 2003.

0100300 Selected Topics in Geometric Group Theory: The mapping class group

Lecture: 4 h, 8 credit points
Tue 11:30-13:00 01.85 Z1, Wed 8:00-9:30 05.20 1C-03
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Prof. Gabriela Weitze-Schmithüsen

Contents

One of the main ideas of geometric group theory is to study finitely generated groups by their action on geometric spaces via symmetries and to link algebraic properties of the groups to geometric properties of the spaces they are acting on.
A main actor in this field is the famous *mapping class group of surfaces*. This is the group of all homeomorphisms modulo those homeomorphisms „which do almost nothing on the surface“, i.e. are homotopic to the identity. This group can be easily defined, but its structure, algebraic properties and geometry give rise to a rich theory and have lead to interesting developments in the research of the last fifty years. Beyond that it still holds several open questions which could not be solved so far.
The study of mapping class groups connects to very different mathematical disciplines. There are purely combinatorial group theoretical aspects of how to present this finitely presentable group. It gets a strongly geometric and combinatorial flavour, if one studies its actions on combinatorial objects as the *curve complex* or the *pants complex* and its relations to so called *braid groups*. Close relations to the theory of low-dimensional manifolds going back to the fundamental work of Thurston, and the study of cohomological stability questions lead deeply into topology. Finally, an object very related to it is *Teichmüller space* on which it acts as its group of automorphisms. If we take the mapping class group of a closed surface of genus g, the quotient of this action is the moduli space M_g, a classification space of closed Riemann surfaces of genus g, which plays an immanent role in algebraic geometry.
As a charming aspect one finds close similarities and analogies between mapping class groups, linear groups and the outer automorphism groups $\mathrm{Out}(F_n)$ which suggests to study these groups in connection. Topics which arise from this are e.g. the study of *congruence groups* and the so called *property T*.
The themes mentioned above span a wide range, much more than we can do in this class.
We will study selected topics from this and start from the scratch by the definition of the mapping class group.
For more information see also: http://www.math.kit.edu/iag3/edu/compabelvar2012w/en

Prerequisites

Basic knowledge in geometry, topology and algebra e.g. from the courses *Grundlagen in Geometrie und Topologie* and *Grundlagen in Algebra und Zahlentheorie*.

References

N. Ivanov: Subgroups of Teichmüller Modular Groups, AMS 1992.
J. Birman: Braids, Linksm And Mapping Class Groups, Princeton University Press 1974.
B. Farb and D. Margalit: A Primer on Mapping Class Groups, Princeton University Press.
B. Farb: Problems on Mapping Class Groups and Related Topics, Proc. Symp. Pure and Applied Math., Volume 74, 2006.

0105800 Brownian Motion

Lecture: 2 h, 4 credit points
Thu 9:45-11:15 5.20 1C-04 (18.10.-7.2.)
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Contents

Brownian motion is one of the most important class of stochastic processes in continuous time and with continuous state space. It has applications in science, engineering and mathematical finance. The object of this course is to give a basic introduction into the theory of Brownian motion. This includes the intersection of Brownian motion with continuous Gaussian processes and continuous Markov processes, the construction and existence of Brownian motion and path properties.

Prerequisites

Probability theory as it is treated in the standard course "Wahrscheinlichkeitstheorie".

References

Durrett, R.: Brownian motion and martingales in analysis, Wadsworth International Group, Belmont, CA, 1984.
Karatzas, I. and Shreve, S. E.: Brownian motion and stochastic calculus, 2nd edn. Springer, New York, 1991.
Klenke, A.: Probability theory, Springer, London, 2008.
Revuz, D. and Yor, M.: Continuous martingales and Brownian motion, 3rd edn. Springer, Berlin, 1999.
Rogers, L. C. G. and Williams, D.: Diffusions, Markov processes, and martingales. Vol. 1, Cambridge University Press, Cambridge, 2000.
Schilling, R. and Partzsch, L.: Brownian Motion : An introduction to stochastic processes, De Gruyter, Berlin, 2012.

0108000 Numerical methods in mathematical finance

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Prof. Tobias Jahnke

Contents

An option is a contract which gives its owner the right to buy or sell an underlying asset at a certain time at a fixed price. The underlying asset is often a stock of a company, and since its value varies in a random way, computing the fair price of the corresponding option is an important and interesting problem which yields a number of mathematical challenges. This lecture provides an introduction to the most important models for option pricing. The main goal, however, is the construction and analysis of numerical methods which approximate the solution of the corresponding differential equations in a stable, accurate and efficient way.
The following topics will be treated:
* Mathematical models for pricing stock options
* Ito integral, Ito formula, stochastic differential equations, Black- Scholes equation
* Binomial methods
* Monte-Carlo methods
* Numerical methods for stochastic differential equations
* Random number generators
* Finite difference methods for parabolic partial differential equations
* Numerical methods for free boundary value problems

The lecture and the exercise classes will be given in English.
Final exam: oral exam, 25-30 minutes
Perspective: A second part of the course will probably be taught in summer 2013.
Diploma and master theses: Since mathematical finance is a rather popular topic, I get a lot of requests to supervise diploma and master theses. Therefore, attending this lecture does not give you a guarantee that I can supervise your thesis.

Prerequisites

Ordinary differential equations and the corresponding numerical methods (cf. lecture "Numerische Methoden für Differentialgleichungen"), and probability theory (cf. lecture "Wahrscheinlichkeitstheorie").
Knowledge about stocks, options, arbitrage, and other aspects from mathematical finance are not required, because the lecture will provide a short introduction to these topics.

0109800 Iterative Methods for Sparse Linear Systems

Lecture: 2 h, 4 credit points
Tue 15:45-17:15 01.93 K2 (16.10.-5.2.)
Dr. Jan Mayer

Contents

Numerical algorithms for finding an approximate solution to some real life problems often yield linear systems having only a few non-zero coefficients in each equation. Such systems are called sparse. They occur in particular for problems having 2D or 3D geometry and whose solution can be described by partial differential equations. The variables describing an approximation of the solution usually depend only on variables that are "closeïn the underlying geometry, but not on any variables that are farther away. As a consequence, only very few variables actually occur in any linear equation. Similarly, any problem that can be described using graphs (e.g. networks, circuits) often results in sparse linear systems. More specifically,sparse linear systems occur in electrical circuit and semiconductor device simulation, fluid dynamics problems, chemical process simulation, macroeconomic modeling and many other applications.
As the coefficient matrix of these systems is large and sparse, typically having no more than 2-100 non-zero elements per row on average and generally having a dimension of at least 100,000 and often much more, simply using Gaussian elimination to solve the system is not feasible. Gaussian elimination does not exploit the large number of zeroes, so that computation time is generally unacceptably high. More importantly, computers often do not have the memory needed to store an entire matrix of this size. Instead, it is usually only possible to store the non-zero elements. As a consequence, we need algorithms for solving a linear system which only make use of these non-zero elements. Iterative methods, requiring only matrix-vector-multiplications with the coefficient matrix, satisfy this condition because all zero elements can be ignored for calculating matrix-vector-products.
In this course, we will cover different iterative methods for solving sparse linear systems. After considering some simple approaches, such as the minimal residual or steepest descent methods, we will discuss more sophisticated projection methods with a special focus on Krylov subspace methods such as GMRES and CG. Special emphasis will be placed on the advantages and disadvantages of each method and on convergence.
The continuation of this course will be offered in the following semester. The emphasis of that course will be on further Krylov space methods and on improving convergence by preconditioning.

Prerequisites

The course requires a good working background in linear algebra. Previous experience with numerical mathematics is helpful but not essential.

References

Axelson, Owe: Iterative Solution Methods.
Meurant, Gerard: Computer Solution Methods for Large Linear Systems.
Saad, Yousef: Iterative Methods for Sparse Linear Systems, 2nd Edition.
van der Vorst, Henk: Iterative Krylov Methods for Large Linear Systems.