INTERNATIONAL PROGRAM (MASTER)

CLASSES: WINTER SEMESTER 2013/2014

0109400 Mathematical Modelling and Simulation
Lecture: 2+1 h, 4 credit points
Fri 9:45-11:15 5.20, 1C-03
Dr. Gudrun Thäter

0106100 Introduction to the Homogenization Theory
Lecture: 2 h, 4 credit points
Tue 8:00-9:30 5.20, 1C-01
Dr. Andrei Khrabustovskyl

0123000 Wavelets
Lecture: 4 h, 8 credit points
Tue 9:45-11:15 5.20, 1C-04, Wed 11:30-13:00 5.20 1C-03
Tutorial: 2 h
Thu 15:45-17:15 1.85, Z2
Prof. Andreas Rieder

0106200 Splitting Methods
Lecture: 2 h, 4 credit points
Mon 11:30-13:00 1.85, Z2
Dr. Katharina Schratz

0105100 Inverse Problems
Lecture: 4 h, 8 credit points
Tue 14:00-15:30 1.85, Z2, Thu 14:00-15:30 1.85, Z2
Tutorial: 2 h;
Mon 15:45-17:15 1.85, Z2
Dr. Frank Hettlich

0113100 p-adic Modular Forms
Lecture: 4 h, 8 credit points
Wed 9:45-11:15 5.20 1C-03, Thu 9:45-11:15 5.20 1C-03
Dr. Fabian Januszewski

0104500 Graph Theory
Lecture: 4 h, 8 credit points
Mon 9:45-11:15 5.20, 1C-03, Wed 8:00-9:30 05.20, 1C-03
Tutorial: 2 h;
Fri 11:30-13:00 1.85, Z1
Prof. Maria Axenovich

0105300 Partial Differential Equations
Lecture: 4 h, 8 credit points
Thu 9:45-11:15 11.40, Tulla, Fri 8:00-9:30 10.81, HS 93
Tutorial: 2 h;
Wed 14:00-15:30 10.91, Redt.
Prof. Tobias Lamm

0118000 Asymptotic Stochastics
Lecture: 4 h, 8 credit points
Tue 11:30-13:00 1.85, Z1, Thu 11:30-13:00 1.85, Z1
Tutorial: 2 h;
Mon 14:00-15:30 1.85, Z1
Prof. Norbert Henze

0124400 Seminar: Statistical Forecasting 6. Sem.
2 h, 3 credit points
Fri 14:00-15:30 5.20, 1C-02
NN

0127100 Seminar: AG Mathematische Physik
2 h, 3 credit points
Mo 11:30-13:00 05.20 1C-01
Prof. Dirk Hundertmark

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30		Hom.Theo.	Graph Theo.		PDE
09:45-11:15	Graph Theo.	Wavelets	Mod.Forms	Mod.Forms PDE	Math.M.Sim.
11:30-13:00	Split.Meth.	Asymp.Stoch.	Wavelets	Asymp.Stoch.	
14:00-15:30		Inv.Prob.		Inv.Prob.	
15:45-17:15					

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: WINTER SEMESTER 2013/2014

0109400 Mathematical Modelling and Simulation

Lecture: 2+1 h, 4 credit points
Fri 9:45-11:15 5.20, 1C-03
Dr. Gudrun Thäter

Contents

The general aim of this lecture course is threefold: to interconnect different mathematical fields, to connect mathematics and real life problems, and to learn to be critical and to ask relevant questions.
We deal with topics such as
Game theory, Oscillation, Population Models, Simulation of traffic, Wiener processes, Chaotic behavior, Heat conduction process, Fluids and flow.
To earn the credits you have to attend the lecture and finish the work on one project during the term in a group of 2 persons. The topic of the project is up to the choice of each group.

The exam can be taken in to ways: Written report of the project to be handed in during the term or oral exam after the term on all matters from the lecture course.

Prerequisites

There are no special prerequisites.

References

Hans-Joachim Bungartz e.a.: Modellbildung und Simulation: Eine Anwendungsorientierte Einführung, Springer, 2009 (German).
Hans-Joachim Bungartz e.a.: Modeling and Simulation: An Application-Oriented Introduction, Springer, Sept. 2013 (English).

0106100 Introduction to the Homogenization Theory

Lecture: 2 h, 4 credit points
Tue 8:00-9:30 5.20, 1C-01
Dr. Andrei Khrabustovskyl

Contents

In many problems of physics and mechanics processes in media with rapidly oscillating spatial local characteristics are studied. There are two main types of such media: composite materials in which the physical processes are described by PDEs with highly oscillating (with respect to spatial variables) coefficients; strongly perforated media in which the physical processes are described by boundary value problems in domains with complicated geometry.

It is practically impossible to solve these problems either by analytical or numerical methods. However when the scale of the microstructure of the medium is much smaller than the scale of the physical process under consideration, the medium has homogenized characteristics (which, in general, differs from local ones). The problem of the homogenization theory is to find these characteristics and using them to construct the homogenized model approximating the initial one and giving global description of the physical process in microinhomogeneous media.

The course devoted to some basic problems and methods of the homogenization theory.
Preliminary plan of the course: Classical problem (homogenization of linear second-order PDE with periodically oscillating coefficients) (Allaire 2002, Cioranescu 1999, Piatniski 2007, Sanchez 1980); 1d example; Formal derivation of the homogenized problem via two-scale asymptotic expansions; Justification of the convergence result.
Some methods: Compensated compactness; G-convergence; Two-scale convergence; Homogenization in perforated domains; Neumann problem; Dirichlet problem. "Strange term"
Other questions: Strongly/weekly connected domains; Homogenization of eigenvalue problems.

Prerequisites

Basic knowledge in functional analysis (Banach and Hilbert spaces, linear operators, strong and weak convergence etc.) and partial differential equations (Sobolev spaces, weak solutions etc.)

References

Allaire G., Homogenization and two-scale convergence , SIAM J. Math. Anal. 23(6) (1992), 1482-1518.
Allaire G., Two-Scale Convergence and Homogenization of Periodic Structures, Lecture notes of the courses held at ICTP, Trieste, September 6–17, 1993
(http://www.mat.uniroma2.it/ braides/ICTP93/ICTP93-Contents.pdf)
Allaire G., Shape optimization by the homogenization method, Springer Verlag, New York, 2002.
Cioranescu D., Donato P., An Introduction to Homogenization, Oxford University Press, New York, 1999.
Cioranescu D., Saint Jean Paulin J., Homogenization of reticulated structures, Springer, New York, 1999.
Marchenko V., Khruslov E., Homogenization of Partial Differential Equation, Birhhauser: Boston, 2006.
Piatnitski A., Chechkin G., Shamaev A., Homogenization. Methods and applications, American Mathematical Society, Providence, 2007.
Sanchez-Palencia E., Nonhomogeneous media and vibration theory, Springer-Verlag, Berlin, 1980.

0123000 Wavelets

Lecture: 4 h, 8 credit points
Tue 9:45-11:15 5.20, 1C-04, Wed 11:30-13:00 5.20 1C-03
Tutorial: 2 h
Thu 15:45-17:15 1.85, Z2
Prof. Andreas Rieder

Contents

Wavelet analysis is a rather new, but meanwhile well established, technique for signal and image processing with various applications in other fields. For instance, the famous JPEG2000 standard for image compression is based upon wavelets.
In this course we will learn the mathematical foundations of wavelet analysis which belong to the field of harmonic analysis. We will motivate wavelet analysis from the shortcomings of Fourier analysis with respect to time frequency representations of signals. Then we will study in detail the properties of the integral wavelet transform. The request for efficient evaluation of the wavelet transform leads to the concept of wavelet bases. Here, we will present the construction of orthogonal and bi-orthogonal wavelet systems. Finally, if time permits, some applications will be discussed: de-noising, image compression, etc.

References (a selection)

Louis, Maaß, Rieder: Wavelets – Theory and Applications, Wiley, 1997
Daubechies: Ten Lectures on Wavelets, SIAM, 1993
Mallat: A Wavelet Tour of Signal Processing. Academic Press, 1997

0106200 Splitting Methods

Lecture: 2 h, 4 credit points
Mon 11:30-13:00 1.85, Z2
Dr. Katharina Schratz

Contents

Due to their computational advantage splitting methods are nowadays omnipresent in scientific computing. They pursue the intention to break down a complicated problem into a series of simpler subproblems. In the context of time integration a common idea is to split up the right-hand side and to decompose the given evolution equation into a sequence of subproblems, which in many situations can be solved far more efficiently or even exactly. The exact solution of the full-problem is then approximated by the composition of the flows associated to the simpler subproblems.
Let us for instance consider the nonlinear Schrödinger equation

$$\begin{aligned} &i\partial_t \psi(t,x) = -\Delta\psi(t,x) + |\psi(t,x)|^2\psi(t,x), \quad x\in\mathbb{T},\ t\in]0,T],\\ &\psi(0,x) = \psi^0(x). \end{aligned} \tag{1}$$

If we decompose the right-hand side into the kinetic and potential part, this leads to the subproblems

$$i\partial_t \psi(t,x) = -\Delta\psi(t,x) \tag{2}$$

and

$$i\partial_t \psi(t,x) = |\psi(t,x)|^2\psi(t,x). \tag{3}$$

The advantage of this splitting ansatz is that both equations can be solved exactly: The kinetic equation (2) by the Fourier decomposition of the solution and the potential equation (3) by noting that the modulus of ψ is constant in time, i.e., $\partial_t|\psi(t,x)|^2 = 0$ in (3). The idea is therefore the following: Instead of solving the full-problem (1) we approximate its solution by the composition of the exact flows of the kinetic and potential equation. Of course some natural questions arise: How good is this approximation, i.e., which order of convergence can we achieve? Are geometric properties (such as the conservation of energy) destroyed by this splitting ansatz? Within this lecture we will address these questions.
First we will investigate the error behavior of splitting methods for ordinary differential equations. The analysis will be based on the Baker-Campbell-Hausdorff formula and the calculus of Lie derivatives. We will in particular discuss splitting methods applied to Hamiltonian systems of ODEs and analyze in how far geometric properties are conserved within this numerical approach. We will then analyze splitting approaches for certain partial differential equations, such as linear Schrödinger equations, Schrödinger equations with a polynomial nonlinearity, as well as the so-called dimension splitting for parabolic evolution equations.
In the exercises we will deepen some theoretical results and carry out practical implementations.

Prerequisites

One should be familiar with basic concepts of the numerical time integration of ODEs and PDEs and functional analysis. A basic knowledge of the theory of semigroups is helpful.

References

Will be given in the lecture.

0105100 Inverse Problems

Lecture: 4 h, 8 credit points
Tue 14:00-15:30 1.85, Z2, Thu 14:00-15:30 1.85, Z2
Tutorial: 2 h;
Mon 15:45-17:15 1.85, Z2
Dr. Frank Hettlich

Contents

In Physical, technical or medical applications there occur often so called inverse problems. These are questions, where the knowledge of measurable data should be used to identify parameter of a given model like in computer tomography. In many cases these problems lead to ill-posed operator equations, i.e. the underlying operator do not have a continuous inverse. The lecture will present the mathematical theory of linear ill-posed equations and will show the phenomenon "ill-posed" by some examples. We will discuss regularization schemes like the Tikhonov regularization in detail for a stable approximation of solutions. Additionally, aspects with respect to non-linear ill-posed problems will be presented. By some quetsions from computer tomography we will enhance these ideas.

Prerequisites

Participants: Students of the mathematical subjects (especially Bachelor of Mathematics in the 5th semester) as well as interested students from physics or engineering sciences. The necessary knowledge from functional analysis will be shown in the course more or less in detail depending on the attending students.

References

H. Engl, M. Hanke, and A. Neubauer, "Regularization of Inverse Problems", Kluwer Academic Publishers Group, Dordrecht, 1996.
A. Kirsch, "An Introduction to the Mathematical Theory of Inverse Problems", Springer-Verlag, New York, 1996.
R. Kress, "Linear Integral Equations", 2nd ed., Springer-Verlag, New York, 1999.
F. Natterer, "The Mathematics of Computerized Tomography", SIAM, Philadelphia, 2001.

0113100 p-adic Modular Forms

Lecture: 4 h, 8 credit points
Wed 9:45-11:15 5.20 1C-03, Thu 9:45-11:15 5.20 1C-03
Dr. Fabian Januszewski

Contents

Modular forms and their p-adic avatars play a central role in modern number theory. Despite being of analytic origin, modular forms contain valuable arithmetic information. A prominent example of this principle is Andrew Wiles' proof of Fermat's Last Theorem. The arithmeticity of modular forms is also reflected in the existence of p-adic modular forms. The latter are more accessible to algebraic methods and are ubiquitious in modern number theory. In contrast to classical modular forms, p-adic modular forms may be deformed p-adically, which allows interesting and powerful arguments.
In particular the close connection between modular forms and Galois representation becomes even closer in the p-adic world. In this class we will learn what a p-adic modular form is, and how we can interpret classical modular forms as p-adic ones. We will see that p-adic modular forms can always be deformed, which has far reaching consequences. In particular it shows that congruences between classical modular forms occur abundantly.
We will focus on the ordinary case, i.e. we will study Hida families. There are two approaches to this theory, one via moduli stacks of elliptic curves and geometric modular forms, and another via cohomology of arithmetic groups. We will follow the latter approach, as it is more easily accessble and requires less prerequisites.

Prerequisites

Preprequisites are the concepts of abstract algebra (group actions, rings, modules, elementary divisor theorem for principal ideal domains), and elementary number theory (congruences and the fundamental properties of p-adic numbers), and basics of topology (as for instance from Ëinführung in Geometrie und Topologie").

References

Hida, Haruzo, Congruences of cusp forms and special values of their zeta functions, Invent. Math., 63, pp. 225-261, 1981, doi=10.1007/BF01393877.
Hida, Haruzo, On congruence divisors of cusp forms as factors of the special values of their zeta functions, Invent. Math., 64, pp. 221-262, 1981, doi=10.1007/BF01389169.
Hida, Haruzo, A p-adic measure attached to the zeta functions associated with two elliptic modular forms. I., Invent. Math., 79, pp. 159-195, 1985, doi=10.1007/BF01388661.
Hida, Haruzo, Iwasawa modules attached to congruences of cusp forms, 1986, 612.10021.
Hida, Haruzo, Galois representations into $\mathrm{GL}_2(\mathbb{Z}_p[[X]])$ attached to ordinary cusp forms, Invent. Math., 85, pp. 545-613, 1986, doi=10.1007/BF01390329.
Hida, Haruzo, Elementary theory of L-functions and Eisenstein series, London Mathematical Society Student Texts. 26, Cambridge University Press. ix, 386 p., 1993.

0104500 Graph Theory

Lecture: 4 h, 8 credit points
Mon 9:45-11:15 5.20, 1C-03, Wed 8:00-9:30 05.20, 1C-03
Tutorial: 2 h;
Fri 11:30-13:00 1.85, Z1
Prof. Maria Axenovich

Contents

The course will be concerned with topics in classical and modern topics in graph theory:
-Properties of trees, cycles, matching, factors
-Forbidden subgraphs
-Planar graphs
-Graph colorings
-Random graphs
-Ramsey theory
-Graph Minors
The class is oriented towards problem solving. The goals of the course for the students is to gain the knowledge about the fundamental concepts in graph theory, solve interesting problems, learn how to write and present the proofs creatively.
The final grade will be based on the written homework and a final exam.

Prerequisites

Basic knowledge of linear algebra, appropriate for students starting from 5th semester.

References

The main source is the book “Graph Theory” by Reinhard Diestel. The English edition can be read for free on the author’s web site.
Additional literature:
D. West: Introduction to graph theory
B. Bollobas: Modern graph theory
A. Bondy and U.S.R. Murty: Graph Theory
L. Lovasz: Combinatorial problems and exercises
G. Chartrand, L. Lesniak and P. Zhang: Graphs and Digraphs

0105300 Partial Differential Equations

Lecture: 4 h, 8 credit points
Thu 9:45-11:15 11.40, Tulla, Fri 8:00-9:30 10.81, HS 93
Tutorial: 2 h;
Wed 14:00-15:30 10.91, Redt.
Prof. Tobias Lamm

Contents

In this introductory lecture to the theory of Partial Differential Equations we start by studying the three model equations: - the Laplace equation
- the heat equation
- the wave equation
in a classical setting. Once this is done we show the existence and regularity of weak solutions of more general elliptic PDE’s in Sobolev spaces.

Prerequisites

Students attending this course should have attended the introductory lectures on Analysis and Linear Algebra. In particular the basic theory of the Lebesgue measure resp. integral and the Theorem of Gauss will be used throughout the course without further explanation.

References

L.C. Evans, Partial Differential Equations, 2nd edition, 2010.
M. Giaquinta and L. Martinazzi, An introduction to the regularity theory for elliptic systems, harmonic maps and minimal graphs, 2nd edition, 2013.
D. Gilbarg and N. Trudinger, Elliptic Partial Differential Equations of second order, 1997.
Q. Han, A basic course in Partial Differential Equations, 2010.
Q. Han and F. Lin, Elliptic Partial Differential Equations, 2nd edition, 2011.
J. Jost, Partial Differential Equations, 3rd edition, 2013.

0118000 Asymptotic Stochastics

Lecture: 4 h, 8 credit points
Tue 11:30-13:00 1.85, Z1, Thu 11:30-13:00 1.85, Z1
Tutorial: 2 h;
Mon 14:00-15:30 1.85, Z1
Prof. Norbert Henze

Contents

Convergence in distribution, method of moments, multivariate normal distribution, characteristic functions and convergence in distribution in $\mathbb{R}^d$, delta method, a Poisson limit theorem for triangular arrays, central limit theorem for m-dependent stationary sequences, Glivenko-Cantelli's theorem, limit theorems for U-statistics, asymptotic properties of maximum likelihood and moment estimators, asymptotic optimality of estimators, asymptotic confidence regions, likelihood ratio tests, weak convergence in metric spaces, Brown Wiener Process, Donsker's theorem, Brownian bridge, goodness-of-fit tests.

Prerequisites

A sound working knowledge in measure-theory based probability theory (especially strong law of large numbers, convergence in distribution in $\mathbb{R}^1$, Central limit theorem of Lindeberg-Lévy), and statistical concepts (tests, confidence regions).

References

Billingsley, P. (1986): Probability and Measure. Wiley, New York.
Billingsley, P. (1968): Convergence of probability measures. Wiley, New York.
Durrett, R. (2010): Probability Theory. Theory and Examples. Fourth Edition. Cambridge University Press.
Ferguson, Th.S. (1996): A Course in Large Sample Theory. Chapman & Hall, London.
Lee, A.J. (1990): U-Statistics. Theory and practice. Marcel Dekker, New York, Basel.
Shao, J. (2003): Mathematical Statistics. Second edition. Springer, New York.