INTERNATIONAL PROGRAM (MASTER)

CLASSES: WINTER SEMESTER 2014/2015

0109400 Mathematical Modelling and Simulation
Lecture: 2 h, 4 credit points
Fri 9:45-11:15 5.20, 1C-03
Tutorial: 2 h
Wed 11:30-13:00 1.85, Z1
Dr. Gudrun Thäter

0106200 Splitting Methods
Lecture: 2 h, 4 credit points
Thu 14:00-15:30 5.20, 1C-02
Tutorial: 2 h
Wed 15:45-17:15 5.20 1C-03
Dr. Katharina Schratz

0118000 Asymptotic Stochastics
Lecture: 4 h, 8 credit points
Tue 14:00-15:30 5.20, 1C-02, Thu 11:30-13:00 1.85, Z1
Tutorial: 2 h;
Mon 15:45-17:15 5.20, 1C-02
Prof. Norbert Henze

0107800 Numerical Methods in Mathematical Finance
Lecture: 4 h, 8 credit points
Thu 8:00-9:30 5.20, 1C-03, Fri 8:00-9:30 5.20 1C-03
Tutorial: 2 h
Mon 14:00-15:30 5.20, 1C-03
Prof. Tobias Jahnke

0104550 Fourier Analysis
Lecture: 4 h, 8 credit points
Mon 11:30-13:00 5.20, 1C-03, Wed 14:00-15:30 5.20 1C-03
Tutorial: 2 h
Tue 15:45-17:15 1.85, Z2
Prof. Maria Girardi

0104800 Functional Analysis
Lecture: 4 h, 8 credit points
Tue 9:45-11:15 Nusselt, Wed 11:30-13:00 Criegee
Tutorial: 2 h
Fri 14:00-15:30 Eiermann
Prof. Roland Schnaubelt

0102650 Forecasting: Theory and Practice I
Lecture: 2 h, 4 credit points
Tue 11:30-13:00 1.85 Z1
Prof. Tilmann Gneiting

0112000 Numerical Methods for Hyperbolic Equations
Lecture: 2 h, 4 credit points
Wed 11:30-13:00 5.20, 1C-02
Tutorial: 2 h
Tue 15:45-17:15 1C-1
Prof. Willy Dörfler

0112700 Particulate Flows
Lecture: 2 h, 4 credit points
Tue 11:30-13:00 5.20 1C-01
Prof. Willy Dörfler

0112700 Algebra
Lecture: 4 h, 8 credit points
Wed 9:45-11:15 AOC 201, Fri 11:30-13:00 NH
Tutorial: 2 h
Thu 9:45-11:15 Chemie-Hörsaal II
Prof. Frank Herrlich

0108000 Homogenization of partial differential equations
Lecture: 3 h, 6 credit points
Wed 15:45-17:15 1.93 K2 (every 2nd week, beg. Oct. 24), Fri 14:00-15:30 1.93 K2
Tutorial: 1 h
Thu 9:45-11:15 5.20 1C-04 (every 2nd week, beg. Nov. 5)
Dr. Andrii Khrabustovskyi

0121300 Seminar: Graph Colouring
2 h, 3 credit points
Wed 11:30-13:00 5.20, 1C-01
Prof. Maria Aksenovich

0124400 Seminar: Statistics
2 h, 3 credit points
Thu 14:00-15:30 05.20 1C-04
Prof. Norbert Henze

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30				Num.Meth.	Num.Meth.
09:45-11:15		Func.Ana.	Algebra		Math.M.Sim.
11:30-13:00	Fourier	Forecast. Part.Flows	Func.Ana. NM.Hyp.Equ.	As.Stoch.	Algebra
14:00-15:30		As.Stoch.	Fourier	Split.M.	Homogen.
15:45-17:15			Homogen.		

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: WINTER SEMESTER 2014/2015

0109400 Mathematical Modelling and Simulation

Lecture: 2+1, 4 credit points
Fri 9:45-11:15 5.20, 1C-03
Dr. Gudrun Thäter

Contents

The general aim of this lecture course is threefold: to interconnect different mathematical fields; to connect mathematics and real life problems and to learn to be critical and to ask relevant questions.
We deal with topics such as Game theory, Oscillation, Population Models, Simulation of traffic, Wiener processes, Chaotic behavior, Heat conduction process, Fluids and flow.

During the lecture course there will be one lecture of a person from industry (Probably someone from ptv group).

To earn the credits you have to attend the lecture and finish the work on one project during the term in a group of 2-3 persons. The topic of the project is up to the choice of each group.

The exam can be taken in three ways: written report of the project to be handed in during the term when there has been an oral presentation as well; oral exam after the term on all matters from the lecture course.

Prerequisites

no special prerequisites

References

Hans-Joachim Bungartz e.a.: Modellbildung und Simulation: Eine Anwendungsorientierte Einführung, Springer, 2009 (German).
Hans-Joachim Bungartz e.a.: Modeling and Simulation: An Application-Oriented Introduction, Springer, Sept. 2013 (English).

0106200 Splitting Methods

Lecture: 2+2, 4 credit points
Thu 14:00-15:30 5.20, 1C-02
Dr. Katharina Schratz

Contents

Due to their computational advantage splitting methods are nowadays omnipresent in scientific computing. They pursue the intention to break down a complicated problem into a series of simpler subproblems. In the context of time integration a common idea is to split up the right-hand side and to decompose the given evolution equation into a sequence of subproblems, which in many situations can be solved far more efficiently or even exactly. The exact solution of the full-problem is then approximated by the composition of the flows associated to the simpler subproblems.
Let us for instance consider the nonlinear Schrödinger equation

$$\begin{aligned} &i\partial_t\psi(t,x) = -\Delta\psi(t,x) + |\psi(t,x)|^2\psi(t,x), \quad x\in\mathbb{T},\, t\in]0,T],\\ &\psi(0,x) = \psi^0(x). \end{aligned} \tag{1}$$

If we decompose the right-hand side into the kinetic and potential part, this leads to the subproblems

$$i\partial_t\psi(t,x) = -\Delta\psi(t,x) \tag{2}$$

and

$$i\partial_t\psi(t,x) = |\psi(t,x)|^2\psi(t,x). \tag{3}$$

The advantage of this splitting ansatz is that both equations can be solved exactly: The kinetic equation (2) by the Fourier decomposition of the solution and the potential equation (3) by noting that the modulus of ψ is constant in time, i.e., $\partial_t|\psi(t,x)|^2 = 0$ in (3). The idea is therefore the following: Instead of solving the full-problem (1) we approximate its solution by the composition of the exact flows of the kinetic and potential equation. Of course some natural questions arise: How good is this approximation, i.e., which order of convergence can we achieve? Are geometric properties (such as the conservation of energy) destroyed by this splitting ansatz? Within this lecture we will address these questions.
Firstly we will investigate the error behavior of splitting methods for ordinary differential equations. The analysis will be based on the Baker-Campbell-Hausdorff formula and the calculus of Lie derivatives. We will in particular discuss splitting methods applied to Hamiltonian systems of ODEs and analyze in how far geometric properties are conserved within this numerical approach. We will then analyze splitting approaches for certain partial differential equations, such as linear Schrödinger equations, Schrödinger equations with a polynomial nonlinearity, as well as the so-called dimension splitting for parabolic evolution equations.
In the exercises we will deepen some theoretical results and carry out practical implementations.

Prerequisites

One should be familiar with basic concepts of the numerical time integration of ODEs and PDEs and functional analysis. A basic knowledge of the theory of semigroups is helpful.

References

Will be given in the lecture.

0118000 Asymptotic Stochastics

Lecture: 4+2, 8 credit points
Tue 14:00-15:30 5.20, 1C-02, Thu 11:30-13:00 1.85, Z1
Prof. Norbert Henze

Contents

Convergence in distribution, method of moments, multivariate normal distribution, characteristic functions and convergence in distribution in R^d, delta method, a Poisson limit theorem for triangular arrays, Central limit theorem for m-dependent stationary sequences, Glivenko-Cantelli's theorem, limit theorems for U-statistics, asymptotic properties of maximum likelihood and moment estimators, asymptotic optimality of estimators, asymptotic confidence regions, likelihood ratio tests, weak convergence in metric spaces, Brown Wiener Process, Donsker's theorem, Brownian bridge, goodness-of-fit tests?

Prerequisites

A sound working knowledge in measure-theory based on probability theory (especially strong law of large numbers, convergence in distribution in R^1, Central limit theorem of Lindeberg-L'ey), and statistical concepts (tests, confidence regions).

References

Billingsley, Patrick (1986): Probability and Measure, Wiley, New York.
Billingsley, Patrick (1968): Convergence of probability measures, Wiley, New York.
Durret, Rick (2010): Probability: Theory and Examples, Cambridge University Press.
Ferguson, Thomas S. (1996): A Course in Large Sample Theory, Chapman and Hall, London.
Lee, A.J. (1990): U-Statistics: theory and practice, Marcel Dekker, New York, Basel.
Shao, Jun (2003): Mathematical Statistics, Springer, New York.

0107800 Numerical Methods in Mathematical Finance

Lecture: 4+2, 8 credit points
Thu 8:00-9:30 5.20, 1C-03, Fri 8:00-9:30 5.20 1C-03
Prof. Tobias Jahnke

Contents

An option is a contract which gives its owner the right to buy or sell an underlying asset at a certain time at a fixed price. The underlying asset is often a stock of a company, and since its value varies randomly, computing the fair price of the corresponding option is an important and interesting problem which yields a number of mathematical challenges. This lecture provides an introduction to the most important models for option pricing. The main goal, however, is the construction and analysis of numerical methods which approximate the solution of the corresponding differential equations in a stable, accurate and efficient way. The following topics will be treated: Mathematical models for pricing stock options; Ito integral, Ito formula, stochastic differential equations, Black-Scholes equation; Binomial methods; Monte-Carlo methods; Numerical methods for stochastic differential equations; Random number generators; Finite difference methods for parabolic partial differential equa-

tions; Numerical methods for free boundary value problems.
The course consists of a lecture and problem classes. Both the lecture and the exercise classes will be given in English. In the problem class the students are supposed to solve small exercises which illustrate the contents of the lecture, and to write short MATLAB programs in order to test and apply the algorithms which will be presented in the lecture. Therefore, half of the problem classes will take place in the computer pool instead of the seminar room. MATLAB skills are required to solve the programming exercises.

Prerequisites

Participants should be familiar with ordinary differential equations and the corresponding numerical methods (cf. lecture "Numerische Methoden für Differentialgleichungen"), probability theory (cf. lecture "Wahrscheinlichkeitstheorie"), and programming in MATLAB. Knowledge about stocks, options, arbitrage and other aspects from mathematical finance are not required, because the lecture will provide a short introduction to these topics.

References

Bingham, Kiesel: Risk-neutral valuation. Pricing and hedging of financial derivatives. 2004, Springer.
Günther, Jüngel: Finanzderivate mit MATLAB. Mathematische Modellierung und numerische Simulation. 2010, Vieweg.
Hanke-Bourgeois: Grundlagen der numerischen Mathematik und des wissenschaftlichen Rechnens. 2009, Vieweg.
Hilber et al.: Computational methods for quantitative finance. Finite element methods for derivative pricing. 2013, Springer.
Seydel: Tools for computational finance. 4th revised and extended ed. 2009, Springer.
Shreve: Stochastic calculus for finance. II: Continuous-time models. 2004, Springer.
Steele: Stochastic calculus and financial applications. 2001, Springer.

0104550 Fourier Analysis

Lecture: 4+2, 8 credit points
Mon 11:30-13:00 5.20, 1C-03, Wed 14:00-15:30 5.20 1C-03
Prof. Maria Girardi

Contents

Fourier series, the Fourier transform on L_1 and L_2, tempered distributions and their Fourier transform, explicit solutions to the heat, Schrodinger, and wave equations, Hilbert transform, Marcinkiewicz's interpolation theorem, singular integral operators, Mihlin's Fourier multiplier theorem.

Prerequisites

One should be familiar with basic concepts of Lebesgue Integration Theory (e.g. Holder's inequality) as well as Functional Analysis (e.g. Hilbert and Banach spaces).

References

As needed and upon request, references will be added as the semester progresses. Here is a start.
Folland: Real analysis. Modern techniques and their applications. Second edition. John Wiley and Sons, Inc., New York, 1999. ISBN: 0-471-31716-0.
Werner: Funktionalanalysis. Third edition. Springer-Verlag, Berlin, 2000. ISBN: 3-540-67645-7.

0104800 Functional Analysis

Lecture: 4+2, 8 credit points
Tue 9:45-11:15 Nusselt, Wed 11:30-13:00 Criegee
Prof. Roland Schnaubelt

Contents

The lecture is concerned with Banach and Hilbert spaces as well as linear operators acting on these spaces. Typical examples are spaces of continuous and integrable functions and linear maps, which one defines via integration of such functions. In this way one can formulate integral equations as affine or linear equations on a suitable Banach space, and one can solve them by means of functional analytic methods. This class of problems was in fact the historical starting point for the development of functional analysis around 1900. In the following years it became a fundamental area of modern analysis and its applications in- and outside of mathematics. A preliminary list of topics: basic properties and examples of metric and Banach spaces and of linear operators, principle of uniform boundedness and open mapping theorem, dual spaces, Hilbert spaces and theorem of Hahn-Banach, weak convergence and theorem of Banach-Alaouglu, Fourier transform, Sobolev spaces, distributions, and applications to partial differential equations.

Prerequisites

Analysis 1-3 and Linear Algebra 1+2.

References

On R. Schnaubelt's webpage one can find the PDF file of the manuscript of his lecture Functional Analysis from winter semester 2011/12. An updated version will be delivered successively during the course of the semester.
H.W. Alt: Lineare Funktionalanalysis, Springer.
J.B. Conway: A Course in Functional Analysis, Springer.
M. Schechter: Principles of Functional Analysis, Academic Press.
A.E. Taylor, D.C. Lay: Introduction to Functional Analysis, Wiley.
D. Werner: Funktionalanalysis, Springer.

0102650 Forecasting: Theory and Practice I

Lecture: 2 h, 4 credit points
Tue 11:30-13:00 1.85 Z1
Prof. Tilmann Gneiting

Contents

A common desire of all humankind is to make predictions for an uncertain future. Clearly then, forecasts ought to be probabilistic, i.e., they ought to take the form of probability distributions over future quantities or events. In this course, which will be continued in summer semester, we will study the probabilistic and statistical foundations of the science of forecasting, along with applications and case studies in economics and meteorology, among other disciplines.
The goal in probabilistic forecasting is to maximize the sharpness of the predictive distributions subject to calibration, based on the information set at hand. We will formalize and study notions of calibration in measure theoretic settings. Proper scoring rules such as the logarithmic score and the continuous ranked probability score serve to assess calibration and sharpness simultaneously, and relate to information theory and convex analysis. As a special case, consistent scoring functions provide decision-theoretically coherent tools for evaluating point forecasts. We will study methodological links to parametric and nonparametric distributional regression techniques, where the goal is to model and to estimate conditional distribution functions. Throughout, we will illustrate concepts and methodologies in data examples, particularly in the context of statistically postprocessed ensemble forecasts of future weather.

Prerequisites

Prerequisites include the basic courses in analysis and linear algebra, an introductory course in probability and statistics (Einführung in die Stochastik or equivalent) and an advanced course in measure theoretic probability (Wahrscheinlichkeitstheorie or equivalent).

References

Gneiting, T. (2008). Editorial: Probabilistic forecasting. Journal of the Royal Statistical Society Series A: Statistics in Society, 171, 319–321.
Gneiting, T. and Katzfuss, M. (2014). Probabilistic forecasting. Annual Review of Statistics and its Application, 1, 125–151.
Non-technical overviews of the topics covered are available in an editorial (Gneiting 2008) and a recent review paper (Gneiting and Katzfuss 2014). Further references will be given in class.

0112000 Numerical Methods for Hyperbolic Equations

Lecture: 2 h, 4 credit points
Wed 11:30-13:00 5.20, 1C-02
Prof. Willy Dörfler

Contents

We present basic theory for equations in conservation form and the fundamental principle to derive numerical methods. As an application we focus on compressible flow equations and Maxwell equations.
Derivation of equations in conservation form, Shocks, Rarefaction waves, weak solutions, Aspects of existence and regularity theory, Discretization of conservation laws with Finite Volume, and Discontinuous Galerkin Methods, Applications.

Oral examination.
Visit the lecture's webpage, linked via `http://www.math.kit.edu/ianm2/~doerfler/` for latest news. Audience: Master Mathematics, Economical Mathematics, Technical Mathematics.

Prerequisites

Numerical methods for Differential Equations.

References

J. S. Hesthaven, T. Warburton: Nodal discontinuous Galerkin methods.
D. Kröner: Numerical Schemes for Conservation Laws.
R. Leveque: Numerical Methods for Conservation Laws.

0112700 Particulate Flows

Lecture: 2 h, 4 credit points
Tue 11:30-13:00 5.20 1C-01
Prof. Willy Dörfler

Contents

We consider several models to mathematically describe flows with particles. Here, we focus on the behaviour of rigid bodies in viscous flows. In addition to the influence of gravity (sedimentation) we consider also the influence of electrical effects.
Derivation of the basic fluid equations (Navier–Stokes and Stokes equations), Models for particulate flows, Stokesian dynamics, Variational methods, Flows with electrical forces, ionic flows (Nernst–Planck equations).
Oral examination.
Audience: Master Mathematics, Economical Mathematics, Technical Mathematics.
Visit the lecture's webpage, linked via `http://www.math.kit.edu/ianm2/~doerfler/` for latest news.

Prerequisites

Numerical methods for Differential Equations. Numerical methods in fluid mechanics.

References

C. Pozrikidis: Introduction to theoretical and computational fluid dynamics (1997).
S. Kim, S. J. Karrila: Microhydrodynamics (2005).

0112700 Algebra

Lecture: 4 h, 8 credit points
Wed 9:45-11:15 AOC 201, Fri 11:30-13:00 NH
Prof. Frank Herrlich

Contents

Field Theory:
Algebraic field extensions, separability, normal extensions, Galois theory.
Roots of unity, cyclotomic fields, solution of polynomial equations by radicals, solvable groups.
Valuation rings:
Valuations, absolute values and norms on rings.
p-adic numbers, nonarchimedean valuations.
Discrete valuation rings, extension of valuations.
Ring theory:
Modules over rings, tensor product of modules, multilinear algebra.
Noetherian rings, ring of polynomials, Hilbert's basis theorem.
Integral ring extensions, integral closure; normalization, Dedekind domains.

Prerequisites

Participants should have basic knowledge about groups, rings and fields as covered by my course „Einführung in Algebra und Zahlentheorie“ (“Introduction to Algebra and Number Theory”), summer term 2014.

References

There are several very classical, but nevertheless very good and useful books on Algebra, notably those by Serge Lang (“Algebra”), Nathan Jacobson (“Basic Algebra I and II”) and Atiyah/MacDonald (“Introduction to Commutative Algebra”). Good textbooks in german are e. g. those by Siegfried Bosch and by Jantzen/Schwermer (both entitled „Algebra“).

0108000 Homogenization of partial differential equations

Lecture: 3 + 1, 6 credit points
Wed 15:45-17:15 1.93 K2 (every 2nd week, beg. Oct. 24), Fri 14:00-15:30 1.93 K2
Dr. Andrii Khrabustovskyi

Contents

In many problems of physics and mechanics processes in media with rapidly oscillating spatial local characteristics are studied. There are two main types of such media: composite materials in which the physical processes are described by PDEs with highly oscillating (with respect to spatial variables) coefficients; strongly perforated media in which the physical processes are described by boundary value problems in domains with complicated geometry. It is practically impossible to solve these problems either by analytical or numerical methods. However when the scale of the microstructure of the medium is much smaller than the scale of the physical process under consideration, the medium has homogenized characteristics (which, in general, differs from local ones). The problem of the homogenization theory is to find these characteristics and using them to construct the homogenized model approximating the initial one and giving global description of the physical process in microinhomogeneous media.
The course devoted to some basic problems and methods of the homogenization theory.

Prerequisites

Basic knowledges in functional analysis (Banach and Hilbert spaces, linear operators, weak and strong convergences etc.) and partial differential equations (Sobolev spaces, weak solutions etc.).

References

G. Allaire: Shape optimization by the homogenization method, Springer, New York, 2002.
V. Marchenko, E. Ya. Khruslov: Homogenization of partial differential equations, Birkhauser, Boston, 2006.
Other references will be given through the lectures.