INTERNATIONAL PROGRAM (MASTER)

CLASSES: WINTER SEMESTER 2015/2016

0123000 Advanced Inverse Problems: Nonlinearity and Banach Spaces
Lecture: 2 h, 4 credit points
Thu 8:00-9:30, 20.30, SR 3.61
Tutorial: 2 h
Mon 9:45-11:15, 20.30, SR 3.61
Prof. Andreas Rieder

0118000 Asymptotic Stochastics
Lecture: 4 h, 8 credit points
Wed 8:00-9:30, 20.30, SR 0.014; Thu 11:30-13:00, 20.30, SR 0.014
Tutorial: 2 h
Fri 9:45-11:15, 20.30, SR 0.014
Prof. Vicky Fasen

0104500 Graph Theory
Lecture: 4 h, 8 credit points
Tue 11:30-13:00, 20.30, SR 1. OG; Thu 9:45-11:15, 20.30, SR 1. OG (2.2.: 20.30, SR 2.58)
Tutorial: 2 h;
Fri 8:00-9:30, 20.30, SR 1. OG
Prof. Maria Aksenovich

0105310 Classical Methods for Partial Differential Equations
Lecture: 4 h, 8 credit points
Mon 11:30-13:00, 20.30, SR 1. OG; Wed 11:30-13:00, 20.30, SR 1. OG
Tutorial: 2 h
Wed 14:00-15:30, 10.91, Redt.
Prof. Michael Plum

0105400 Travelling Waves
Lecture: 3 h, 6 credit points
Tue 15:45-17:15, 20.30, SR 3.68
Tutorial: 1 h
Thu 15:45-17:15, 20.30, SR 3.68
JProf. Rottmann-Matthes

011500 Algebraic Topology II
Lecture: 4 h, 8 credit points
Mon 15:45-17:15, 20.30, -1.012; Tue 14:00-15:30, 20.30, -1.011
Tutorial: 2 h
Fri 14:00-15:30. 20.30, SR 3.69
Prof. Roman Sauer

0102300 L2-Invariants
Lecture: 2 h, 4 credit points
Thu 15:45-17:15, 20.30, SR 2.58
Tutorial: 2 h
Tue 9:45-11:15. 20.30, SR 2.58
Dr. Holger Kammeyer

0110300 Finite Element Methods
Lecture: 4 h, 8 credit points
Tue 9:45-11:15, 20.20, SR -1.025; Thu 14:00-15:30, 20.30, SR 1. OG
Tutorial: 2 h
Wed 9:45-11:15, 20.30, SR 3.61
Prof. Marlis Hochbruck

0124350 Seminar (Statistical Forecasting)
Tue 14:00-15:30, 20.30, SR 2.59
Prof. Tilmann Gneiting

0126400 Seminar (Aspects of Numerical Time Integration)
to be announced
JProf. Katharina Schratz

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30			Asymp. Stoch.	Adv. Inv. Probl.	
09:45-11:15		Fin. El. Meth.		Graph Th.	
11:30-13:00	Class. Meth. PDE	Graph Th.	Class. Meth. PDE	Asymp. Stoch.	
14:00-15:30		Algebr. Top. II		Fin. El .Meth.	
15:45-17:15	Algebr. Top. II	Trav. Waves		L2-Inv.	

GERMAN CLASSES

Optional German language classes should be attended in the late afternoon and evening.

CLASSES: WINTER SEMESTER 2015/2016

0123000 Advanced Inverse Problems: Nonlinearity and Banach Spaces

Lecture: 2, 4 credit points
Thu 8:00-9:30, 20.30, SR 3.61
Tutorial: 2 h
Mon 9:45-11:15, 20.30, SR 3.61
Prof. Andreas Rieder

Contents

The main topic of the course is to solve nonlinear ill-posed problems

$$F(x) = y$$

where $F\colon D(F) \subset X \to Y$ operates between the Banach spaces X and Y with domain of definition $D(F)$. This kind of inverse problem gained a lot of interest over the last years because several applications and constraints are formulated quite naturally in a Banach space framework: sparsity, uniform and impulsive noise, preserving discontinuities (edges) etc.
In parameter identification tasks, for instance, the searched-for parameter often appears in the governing partial differential equations as an L^∞-coefficient, e.g., electrical impedance tomography.
The following topics are intended to be covered:
Hilbert spaces: Inexact Newton solvers for nonlinear ill-posed Problems with application to Electrical Impedance Tomography
Banach spaces: The method of Approximate Inverse; Geometry of Banach spaces; Gradient-like iterations for linear Problems; Tikhonov-Phillips Regularization; Inexact Newton solvers;

Prerequisites

Knowledge of Functional Analysis and of some basics in Inverse Problems are advantageous.

References

T. Schuster, B. Kaltenbacher, B. Hofmann, K. S. Kazimierski: Regularization methods in Banach spaces, de Gruyter, 2012.
B. Kaltenbacher, A. Neubauer, O. Scherzer: Iterative Regularization Methods for Nonlinear Ill-Posed Problems, de Gruyter, 2008.

0118000 Asymptotic Stochastics

Lecture: 4 h, 8 credit points
Wed 8:00-9:30, 20.30, SR 0.014; Thu 11:30-13:00, 20.30, SR 0.014
Tutorial: 2 h
Fri 9:45-11:15, 20.30, SR 0.014
Prof. Vicky Fasen

Contents

Topics:
Convergence in distribution
method of moments
multivariate normal distribution
characteristic functions and convergence in distribution in R^d
delta method
a Poisson limit theorem for triangular arrays
Central limit theorem for m-dependent stationary sequences
Glivenko-Cantelli's theorem
limit theorems for U-statistics
asymptotic properties of maximum likelihood and moment estimators
asymptotic optimality of estimators
asymptotic confidence regions
likelihood ratio tests
weak convergence in metric spaces
Brownian motion
Donsker's theorem
Brownian bridge
goodness-of-fit tests.

Prerequisites

A sound working knowledge in measure-theory based on probability theory (especially strong law of large numbers, convergence in distribution in R^1, Central limit theorem of Lindeberg-Lévy), and statistical concepts (tests, confidence regions).

References

Billingsley, P. (1986): Probability and Measure. Wiley, New York.
Billingsley, P. (1968): Convergence of probability measures. Wiley, New York.
Durrett, R. (2010): Probability Theory. Theory and Examples. Fourth Edition. Cambridge University Press.
Ferguson, Th.S. (1996): A Course in Large Sample Theory. Chapman and Hall, London.
Lee, A.J. (1990): U-Statistics. Theory and practice. Marcel Dekker, New York, Basel.
Shao, J. (2003): Mathematical Statistics. Second edition. Springer, New York.
van der Vaart, A.W. (2007): Asymptotic Statistics. Cambridge Univ. Press, Cambridge.

0104500 Graph Theory

Lecture: 4 h, 8 credit points
Tue 11:30-13:00, 20.30, SR 1. OG; Thu 9:45-11:15, 20.30, SR 1. OG (2.2.: 20.30, SR 2.58)
Tutorial: 2 h;
Fri 8:00-9:30, 20.30, SR 1. OG
Prof. Maria Aksenovich

Contents

The course will be concerned with topics in classical and modern graph theory:
-Properties of trees, cycles, matchings, factors, connectivity
-Forbidden subgraphs
-Planar graphs
-Graph colorings
-Random graphs
-Ramsey theory
-Graph Minors
The class is oriented towards problem solving. The goals of the course for the students is to gain the knowledge about the fundamental concepts in graph theory, to solve interesting problems, and to learn how to write and present the proofs creatively.

The final grade will be based on the written exam. The bonus points will be given for weekly or biweekly homework assignments.

Prerequisites

Basic knowledge of linear algebra.

References

The main source is the book Graph Theory by Reinhard Diestel. The English edition can be read for free on the author's web site (http://diestel-graph-theory.com/).
Additional literature
D. West: Introduction to graph theory.
B. Bollobas: Modern graph theory.
A. Bondy and U.S.R. Murty: Graph Theory.
L. Lovasz: Combinatorial problems and exercises.
G. Chartrand, L. Lesniak and P. Zhang: Graphs and Digraphs.

0105310 Classical Methods for Partial Differential Equations

Lecture: 4 h, 8 credit points
Mon 11:30-13:00, 20.30, SR 1. OG; Wed 11:30-13:00, 20.30, SR 1. OG
Tutorial: 2 h
Wed 14:00-15:30, 10.91, Redt.
Prof. Michael Plum

Contents

A differential equation is a relation between an unknown function (to be determined) and its derivatives. While for ordinary differential equations the unknown function depends on a single independent variable, it depends on several variables for partial differential equations. A huge variety of processes in science and technology is described by partial differential equations, which therefore belong to the most important objects of investigation in Applied Mathematics.
The number of phenomena occurring in the context of partial differential equations, and the number of methods and techniques to investigate them, is by far too complex to be the content of a one semester course. The lecture course can therefore only be of an introductory type. Topics to be treated are e.g. the classical wave-, Poisson-, and heat equation, maximum principles, separation of variables, classification of quasilinear second-order equations and first-order systems, normal forms, a fixed-point approach for second-order hyperbolic equations. Strong emphasis will be put on many examples from physics and engineering.

The lectures will be accompanied by exercise lessons. Attendance of these exercises is strongly recommended to all participants.

As already mentioned, this lecture course can cover only a small portion of the overall topic of partial differential equations. Deeper knowledge can be aquired in further subsequent courses.

Prerequisites

The lecture course addresses students in their fifth semester (third year) or higher, with substantial knowledge in analysis and linear algebra. It is suitable for students of mathematics, and for students of other subjects who have strong mathematical interests.

References

Evans, L. C.: Partial Differential Equations. Graduate Studies in Mathematics 19, American Mathematical Society.
Copson, E. T.: Partial Differential Equations; Cambridge University Press.
Courant, R., Hilbert, D.: Methods of Mathematical Physics, Vol. I + II; Wiley Classics.
Hellwig, G.: Partial Differential Equations; Teubner.

For those who understand German (or want to learn it):
Hellwig, G.: Partielle Differentialgleichungen; Teubner.
Leis, R.: Vorlesungen über partielle Differentialgleichungen zweiter Ordnung; Bibliographisches Institut, Mannheim.

0105400 Travelling Waves

Lecture: 3 h, 6 credit points
Tue 15:45-17:15, 20.30, SR 3.68
Tutorial: 1 h
Thu 15:45-17:15, 20.30, SR 3.68
JProf. Rottmann-Matthes

Contents

In this lecture we will consider traveling wave solutions to partial differential equations in $1+1$-dimensions. These are solutions $u : \mathbb{R} \times [0, \infty) \to \mathbb{R}^m$ of the form $u(x,t) = U(x-ct)$, to a system of reaction-diffusion equations $u_t = u_{xx} + f(u)$. Here $U : \mathbb{R} \to \mathbb{R}^m$ denotes the profile and c denotes the velocity of the wave.
First we will consider the question of existence of such solutions. After that we will look at the question of stability, i.e. whether a small perturbation of a traveling wave solution converges to the traveling wave solution as time tends to ∞. Since the equation is space independent, one can only expect stability with asymptotic phase: $\lim_{t\to\infty} u(\cdot,t) - U(\varphi_\infty - ct) = 0$, for some suitable φ_∞.

Prerequisites

Good knowledge of Analysis I-III, good knowledge of Linear Algebra, some knowledge of Functional Analysis, some knowledge of Spectral theory, some knowledge of PDEs.
Furthermore, Bochner-Integration might be helpful but is not necessary.

References

W. Arendt, Charles J. K. Batty, M. Hieber, and F. Neubrander: Vector-valued Laplace transforms and Cauchy problems. Volume 96 of Monographs in Mathematics, Birkhäuser Verlag, Basel, 2001.
D. G. Aronson and H. F. Weinberger: Nonlinear diffusion in population genetics, combustion, and nerve pulse propagation. In: Partial differential equations and related topics (Program, Tulane Univ., New Orleans, La., 1974), volume 446 of Lecture Notes in Mathematics, pages 5–49. Springer, Berlin, 1975.
L.C. Evans: Partial differential equations. Volume 19 of Graduate Studies in Mathematics, AMS, Providence, RI, 1998.
Roger Knobel: An introduction to the mathematical theory of waves. Providence, RI: American Mathematical Society, 2000.
Heinz-Otto Kreiss and Jens Lorenz: Initial-boundary value problems and the Navier-Stokes equations. Volume 136 of Pure and Applied Mathematics, Academic Press Inc., Boston, MA, 1989.
Björn Sandstede: Stability of travelling waves. Handbook of dynamical systems, Vol. 2, pages 983–1055, North-Holland, Amsterdam, 2002.

011500 Algebraic Topology II

Lecture: 4 h, 8 credit points
Mon 15:45-17:15, 20.30, -1.012; Tue 14:00-15:30, 20.30, -1.011
Tutorial: 2 h
Fri 14:00-15:30. 20.30, SR 3.69
Prof. Roman Sauer

Contents

The course covers Lefschetz numbers, singular cohomology, product structures on cohomology and Poincare duality. We also discuss the geometric interpretation of the cup product as intersection numbers of submanifolds. If time permits, we give an introduction to basic homotopy theory as well.
There will be oral exams of about 25 min after the end of the course.

Prerequisites

Students are expected to be familiar with the contents of the course *Algebraic Topology*, i.e. with computations of fundamental groups via van-Kampen's theorem, axiomatic homology theory, CW complexes, and the computation of homology via cellular homology. The teaching concept is a mixture of an inquiry-based approach and a traditional lecture approach. This means that active participation is of utmost importance.

References

G. E. Bredon: Topology and geometry, Graduate Texts in Mathematics, 139, Springer-Verlag, 1997, xiv+557.
T. Dieck: Algebraic topology, EMS Textbooks in Mathematics, EMS, Zürich, 2008, xii+567.
A. Hatcher: Algebraic topology, Cambridge University Press, 2002, xii+544. (available under `http://www.math.cornell.edu/ hatcher/AT/ATpage.html`).
J. P. May: A concise course in algebraic topology, Chicago Lectures in Mathematics, University of Chicago Press, 1999, x+243.

0102300 L2-Invariants

Lecture: 2 h, 4 credit points
Thu 15:45-17:15, 20.30, SR 2.58
Tutorial: 2 h
Tue 9:45-11:15. 20.30, SR 2.58
Dr. Holger Kammeyer

Contents

Classical algebraic topology is concerned with invariants such as Betti numbers, Euler characteristic and Reidemeister torsion of compact spaces. It is a fruitful idea to try and find counterparts to these invariants on the universal covering. The latter, however, is no longer compact unless the fundamental group is finite. Technically, this has the effect that common associated algebraic structures, like homology and chain complexes, might be infinite-dimensional and do not allow an ad hoc definiton of useful invariants.

The remedy to this dilemma is passing to the "L^2-completion" to obtain Hilbert spaces. For these objects powerful tools from analysis give a means to define *L^2-invariants* which extract valuable and otherwise invisible information. L^2-invariants form an area of active research and have proven their usefulness in contexts as diverse as group theory, differential geometry, ergodic theory, K-theory and more recently also knot theory and quantum groups.

- Hilbert modules and von Neumann dimension
- L^2-Betti numbers of CW complexes and groups
- Novikov–Shubin invariants
- Fuglede–Kadison determinant and L^2-torsion

Prerequisites

Concepts from "Introduction to Geometry and Topology" (fundamental group and covering spaces) as well as "Algebraic Topology" (CW complexes, chain complexes, cellular homology) will be used. I intend to explain methods from other fields when needed.

References

W. Lück, L^2-invariants: theory and applications to geometry and K-theory, Erg. Math. Grenzgeb. (3), vol. 44, Springer-Verlag, Berlin, 2002.

0110300 Finite Element Methods

Lecture: 4 h, 8 credit points
Tue 9:45-11:15, 20.20, SR -1.025; Thu 14:00-15:30, 20.30, SR 1.067
Tutorial: 2 h
Wed 9:45-11:15, 20.30, SR 3.61
Prof. Marlis Hochbruck

Contents

This lecture provides an introduction to the theory of finite element methods for elliptic boundary value problems in $\mathbb{R}^n$. In particular, stability and convergence will be proved and concepts for the implementation of such methods will be explained. Moreover, the numerical solution of elliptic eigenvalue problems and mixed methods for saddle point problems will be investigated.

Prerequisites

The students are expected to be familiar with the basics of numerical analysis, in particular interpolation, numerical integration, solution of linear systems and eigenvalue problems. Some basic knowledge in functional analysis and the analysis of boundary value problem is helpful but the main results will be repeated in the lecture.

References

M. Hochbruck: Lecture Notes, will be provided for free on the course web page.
S. Brenner, R. Scott: The Mathematical Theory of Finite Element Methods, Springer, 2008.
D. Braess: Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics. Springer, 2007.