INTERNATIONAL PROGRAM (MASTER)

CLASSES: WINTER SEMESTER 2016/2017

0109400 Mathematical Modelling and Simulation
Lecture: 2 h, 4 credit points
Fri 9:45-11:15, 30.41, HS III
Tutorial: 1 h
a project which translates into about 45 min per week (during term)
Dr. Gudrun Thäter

0106200 Splitting Methods
Lecture: 2 h, 4 credit points
Tue 11:30-13:00, 20.30, SR 3.68
Tutorial: 2 h
Thu 15:45-17:15, 20.30, SR 2.67
JProf. Katharina Schratz

0118000 Asymptotic Stochastics
Lecture: 4 h, 8 credit points
Tue 8:00-9:30, 20.30, SR 1.067; Thu 11:30-13:00, 20.40, HS 9
Tutorial: 2 h;
Fri 9:45-11:15, 20.30, SR 0.014
Prof. Norbert Henze

0107800 Numerical Methods in Mathematical Finance
Lecture: 4 h, 8 credit points
Mon 8:00-9:30, 20.30, SR 0.014; Thu 8:00-9:30, 20.30, SR 0.014
Tutorial: 2 h
Wed 14:00-15:30, 20.30, SR 3.69
Prof. Tobias Jahnke

0103650 Statistical Forecasting I
Lecture: 2 h, 4 credit points
Tue 14:00-15:30, 20.30, SR 2.58
Prof. Tilmann Gneiting

0111500 Algebraic Topology II
Lecture: 4 h, 8 credit points
Mon 11:30-13:00, 20.30, -1.012; Wed 11:30-13:00, 20.30, -1.011
Tutorial: 2 h
Thu 14:00-15:30, 20.30, SR 3.69
Dr. Caterina Campagnolo

0110300 Finite Element Methods
Lecture: 4 h, 8 credit points
Wed 8:00-9:30, 20.30, SR 0.014; Fri 8:00-9:30, 20.30, SR 1.067
Tutorial: 2 h
Mon 11:30-13:00, 20.30, SR 3.61
Prof. Tobias Jahnke

0104800 Functional Analysis
Lecture: 4 h, 8 credit points
Mon 9:45-11:15, 20.30, SR 1.067; Thu 9:45-11:15, 20.30, SR 1.067
Tutorial: 2 h
Fri 14:00-15:30, 20.40, Eiermann
Prof. Tobias Lamm

0109200 Numerical Methods for Maxwell's Equations
Lecture: 2 h, 6 credit points
Wed 9:45-11:15, 20.30, SR 3.01; Thu 9:45-11:15, 20.30, SR 3.61
(one of them will become the regular date for the lecture, the other an alternative date, this will be discussed in the first class on Wed, Oct 19)
Tutorial: 2 h
Mon 15:45-17:15, 20.30, SR 3.61
(this slot might also be used for the lecture, if necessary)
Prof. Marlis Hochbruck

0150300 Combinatorics in the Plane
Lecture: 3 h, 7 credit points
Tue 11:30-13:00, 20.30, SR 3.61
Tutorial: 2 h
Wed 14:00-15:30, 20.30, SR 2.59
Dr. Torsten Ueckerdt

0104600 Nonlinear Boundary Value Problems
Lecture: 4 h, 8 credit points
Tue 15:45-17:15, 20.30, SR 3.68; Fri 11:30-13:00, 20.30, SR 3.68
Tutorial: 2 h
Wed 15:45-17:15, 20.30, SR 3.68
Prof. Michael Plum

Time-table for lectures

	Monday	Tuesday	Wednesday	Thursday	Friday
08:00-09:30	Num.Meth.F.	As.Stoch.	Fin.El.Meth.	Num.Meth.F.	Fin.El.Meth.
09:45-11:15	Func.Ana.		Num.Meth.Max.	Func.Ana. Num.Meth.Max.	Math.Mod.
11:30-13:00	Alg.Top.II	Combinatorics Splitt.Meth.	Alg.Top.II	As.Stoch.	Nonlin.BVP
14:00-15:30		Stat.Forec.I			
15:45-17:15		Nonlin.BVP			

GERMAN CLASSES
Optional German language classes should be attended in the late afternoon and evening.

CLASSES: WINTER SEMESTER 2015/2016

0109400 Mathematical Modelling and Simulation

Lecture: 2, 4 credit points
Fri 9:45-11:15, 30.41, HS III
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a project which translates into about 45 min per week (during term)
Dr. Gudrun Thäter

Contents

The general aim of this lecture course is threefold: To interconnect different mathematical fields; To connect mathematics with real life problems and students of different lecture courses; To learn to be critical and to ask relevant questions.
There are no special prerequisites. We deal with topics such as: Game theory, Oscillation, Population Models, Simulation of traffic, Wiener processes, Chaotic behavior, Heat conduction process, Fluids and flow.
During the lecture course there will be one lecture of a person from industry and an excursion at the end.
To earn the credits you have to finish the work on one project during the term in a group of 2-3 persons. The topic of the project is up to the choice of each group. There will be recommended projects for groups without own ideas.
The project can be finished either with a written report to be handed in one week before the end of the term or an oral presentation during the two last weeks of the term.
The **exam** is an oral exam on all matters from the lecture course. There will be two examination days: The first one a week or two after the term and the second one immediately before the new term.

References

Hans-Joachim Bungartz e.a.: Modellbildung und Simulation: Eine Anwendungsorientierte Einführung, Springer, 2009 (German).
Hans-Joachim Bungartz e.a.: Modeling and Simulation: An Application-Oriented Introduction, Springer, Sept. 2013 (English).
Both books are available as e-books through the KIT-library!
Please find further information at
`http://www.math.kit.edu/ianm2/lehre/semmathmodel2016w/en`

0106200 Splitting Methods

Lecture: 4 h, 4 credit points
Tue 11:30-13:00, 20.30, SR 3.68
Tutorial: 2 h
Thu 15:45-17:15, 20.30, SR 2.67
JProf. Katharina Schratz

Contents

Due to their computational advantage splitting methods are nowadays omnipresent in scientific computing. They pursue the intention to break down a complicated problem into a series of simpler subproblems. In the context of time integration a common idea is to split up the right-hand side and to decompose the given evolution equation into a sequence of subproblems, which in many situations can be solved far more efficiently or even exactly. The exact solution of the full-problem is then approximated by the composition of the flows associated to the simpler subproblems.
Let us for instance consider the nonlinear Schrödinger equation

$$\begin{aligned} &i\partial_t\psi(t,x) = -\Delta\psi(t,x) + |\psi(t,x)|^2\psi(t,x), \quad x \in \mathbb{T},\ t \in]0,T], \\ &\psi(0,x) = \psi^0(x). \end{aligned} \tag{1}$$

If we decompose the right-hand side into the kinetic and potential part, this leads to the subproblems

$$i\partial_t\psi(t,x) = -\Delta\psi(t,x) \tag{2}$$

and

$$i\partial_t\psi(t,x) = |\psi(t,x)|^2\psi(t,x). \tag{3}$$

The advantage of this splitting ansatz is that both equations can be solved exactly: The kinetic equation (2) by the Fourier decomposition of the solution and the potential equation (3) by noting that the modulus of ψ is constant in time, i.e., $\partial_t|\psi(t,x)|^2 = 0$ in (3). The idea is therefore the following: Instead of solving the full-problem (1) we approximate its solution by the composition of the exact flows of the kinetic and potential equation. Of course some natural questions arise: How good is this approximation, i.e., which order of convergence can we achieve? Are geometric properties (such as the conservation of energy) destroyed by this splitting ansatz? Within this lecture we will address these questions.
Firstly we will investigate the error behavior of splitting methods for ordinary differential equations. The analysis will be based on the Baker-Campbell-Hausdorff formula and the calculus of Lie derivatives. We will in particular discuss splitting methods applied to Hamiltonian systems of ODEs and analyze in how far geometric properties are conserved within this

numerical approach. We will then analyze splitting approaches for certain partial differential equations, such as linear Schrödinger equations, Schrödinger equations with a polynomial nonlinearity, as well as the so-called dimension splitting for parabolic evolution equations. In the exercises we will deepen some theoretical results and carry out practical implement

Prerequisites

One should be familiar with basic concepts of the numerical time integration of ODEs and PDEs and functional analysis. A basic knowledge of the theory of semigroups is helpful.

References

Will be given in the lecture.

0118000 Asymptotic Stochastics

Lecture: 4 h, 8 credit points
Tue 8:00-9:30, 20.30, SR 1.067; Thu 11:30-13:00, 20.40, HS 9
The course will start October 18*th*
Tutorial: 2 h;
Fri 9:45-11:15, 20.30, SR 0.014
Prof. Norbert Henze

Contents

Convergence in distribution, method of moments, multivariate normal distribution, characteristic functions and convergence in distribution in $\mathbb{R}^d$, delta method, a Poisson limit theorem for triangular arrays, Central limit theorem for m-dependent stationary sequences, Glivenko-Cantelli's theorem, limit theorems for U-statistics, asymptotic properties of maximum likelihood and moment estimators, asymptotic optimality of estimators, asymptotic confidence regions, likelihood ratio tests, weak convergence in metric spaces, Brown Wiener Process, Donsker's theorem, Brownian bridge, goodness-of-fit tests

Prerequisites

A sound working knowledge in measure-theory based probability theory (especially strong law of large numbers, convergence in distribution in $\mathbb{R}^1$, Central limit theorem of Lindeberg–Feller)), conditional expectations, and statistical concepts (tests, confidence regions)

References

Billingsley, P. (1986): Probability and Measure. Wiley, New York.
Billingsley, P. (1999): Convergence of probability measures, Second edition. Wiley, New York.
Durrett, R. (2010): Probability Theory. Theory and Examples. Fourth Edition. Cambridge University Press.
Ferguson, Th.S. (1996): A Course in Large Sample Theory. Chapman & Hall, London.
Lee, A.J. (1990): U-Statistics. Theory and practice. Marcel Dekker, New York, Basel.
Shao, J. (2003): Mathematical Statistics. Second edition. Springer, New York.

0107800 Numerical Methods in Mathematical Finance

Lecture: 4 h, 8 credit points
Mon 8:00-9:30, 20.30, SR 0.014; Thu 8:00-9:30, 20.30, SR 0.014
Tutorial: 2 h
Wed 14:00-15:30, 20.30, SR 3.69
Prof. Tobias Jahnke

Contents

An option is a contract which gives its owner the right to buy or sell an underlying asset at a certain time at a fixed price. The underlying asset is often a stock of a company, and since its value varies randomly, computing the fair price of the corresponding option is an important and interesting problem which yields a number of mathematical challenges. This lecture provides an introduction to the most important models for option pricing. The main goal, however, is the construction and analysis of numerical methods which approximate the solution of the corresponding differential equations in a stable, accurate and efficient way.
The following topics will be treated:
Mathematical models for pricing stock options, Itô integral, Itô formula, stochastic differential equations, Black-Scholes equation, Binomial methods, Monte-Carlo methods, Numerical methods for stochastic differential equations, Random number generators, Finite difference methods for parabolic partial differential equations, Numerical methods for free boundary value problems.
The course consists of a lecture and a problem class, both given in English. In the problem class the students will solve small exercises which illustrate the contents of the lecture. Moreover, participants are supposed to write short MATLAB programs in order to test and apply the algorithms which will be presented in the lecture.
A second part of the course will be taught in summer 2017.

Prerequisites

Participants have to be familiar with
ordinary differential equations and the corresponding numerical methods (cf. lecture "Numerische Methoden für Differentialgleichungen"), probability theory (cf. lecture "Wahrscheinlichkeitstheorie"), and programming in MATLAB.
Knowledge about stocks, options, arbitrage and other aspects from mathematical finance are not required, because the lecture will provide a short introduction to these topics.

References

N. H. Bingham, R. Kiesel: Risk-neutral valuation. Pricing and hedging of financial derivatives, Springer 2004.
M. Günther, A. Jüngel: Finanzderivate mit MATLAB. Mathematische Modellierung und numerische Simulation. Vieweg 2nd ed. 2010.
M. Hanke-Bourgeois: Grundlagen der numerischen Mathematik und des wissenschaftlichen Rechnens, 3rd revised ed., Vieweg+Teubner 2009.
N. Hilber, O. Reichmann, C. Schwab, C. Winter: Computational methods for quantitative finance. Finite element methods for derivative pricing, Springer finance 2013.

R. Seydel: Tools for computational finance. 4th revised and extended ed., Springer 2009.
S. Shreve: Stochastic calculus for finance. II: Continuous-time models. Springer Finance 2004.
J. M. Steele: Stochastic calculus and financial applications. 45, Applic. of Math., NY, Springer 2001.

0103650 Statistical Forecasting I

Lecture: 2 h, 4 credit points
Tue 14:00-15:30, 20.30, SR 2.58
Prof. Tilmann Gneiting

Contents

IA common desire of all humankind is to make predictions for the future. As the future is inherently uncertain, forecasts ought to be probabilistic, i.e., they ought to take the form of probability distributions over future quantities or events. In this course, which comprises Part I of a two semester series, we will study the probabilistic and statistical foundations of the science of forecasting.
The goal in probabilistic forecasting is to maximize the sharpness of the predictive distributions subject to calibration, based on the information set at hand. Proper scoring rules such as the logarithmic score and the continuous ranked probability score serve to assess calibration and sharpness simultaneously, and relate to information theory and convex analysis. As a special case, consistent scoring functions provide decision-theoretically coherent tools for evaluating point forecasts. Throughout, concepts and methodologies will be illustrated in data examples and case studies.
There will be an oral exam (30 minutes) covering both Part I and Part II at dates announced toward the end of summer semester [MATHST28: 8 ECTS in total].

Prerequisites

A firm understanding of the contents of module Probability Theory [MATHBAST02] is essential.

References

Non-technical overviews of the topics covered are available in an editorial (Gneiting 2008) and a recent review paper (Gneiting and Katzfuss 2014). Technical references include the papers by Gneiting and Raftery (2007), Gneiting (2011) and Gneiting and Ranjan (2013).
Gneiting, T. (2008). Editorial: Probabilistic forecasting. *Journal of the Royal Statistical Society Series A: Statistics in Society*, **171**, 319–321.
Gneiting, T. (2011). Making and evaluating point forecasts. *Journal of the American Statistical Association*, **106**, 746–762.
Gneiting, T. and Katzfuss, M. (2014). Probabilistic forecasting. *Annual Review of Statistics and its Application*, **1**, 125–151.
Gneiting, T. and Raftery, A. E. (2007). Strictly proper scoring rules, prediction, and estimation. *Journal of the American Statistical Association*, **102**, 359–378.
Gneiting, T. and Ranjan, R. (2013). Combining predictive distributions. *Electronic Journal of Statistics*, **7**, 1747–1782.

011500 Algebraic Topology II

Lecture: 4 h, 8 credit points
Mon 11:30-13:00, 20.30, -1.012; Wed 11:30-13:00, 20.30, -1.011
Tutorial: 2 h
Thu 14:00-15:30, 20.30, SR 3.69
Dr. Caterina Campagnolo

Contents

At the beginner's level, algebraic topology separates naturally into the two broad topics of homology and homotopy. The course covers the essentials of singular homology, including the axiomatic approach and the computational approach via cellular complexes. Basic properties of homotopy groups are discussed, but the focus is mostly on homology. Fundamental ideas of homological algebra will be an important part of the course. A highlight at the end of the course is the effective computation of homology groups of cell complexes.
The final grade will be determined by a written exam (120 min) after the end of the course.

Prerequisites

Students are expected to be familiar with basic notions of set-theoretic topology like topological space, quotient topology, or manifold and with covering theory. On the algebraic side students are only expected to know some basic notions like groups and modules.

References

G. E. Bredon: Topology and geometry, Graduate Texts in Mathematics, 139, Springer NY, 1997, pp. xiv+557.
T. tom Dieck: Algebraic topology, EMS Textbooks in Mathematics, European Mathematical Society (EMS), Zürich, 2008, pp. xii+567.
A. Hatcher: Algebraic topology, Cambridge University Press, 2002, p. xii+544; available under `http://www.math.cornell.edu/ hatcher/AT/ATpage.html`.
J. P. May: A concise course in algebraic topology, Chicago Lectures in Mathematics, University of Chicago Press, 1999, pp. x+243.

0110300 Finite Element Methods

Lecture: 4 h, 8 credit points
Wed 8:00-9:30, 20.30, SR 0.014; Fri 8:00-9:30, 20.30, SR 1.067
Tutorial: 2 h
Mon 11:30-13:00, 20.30, SR 3.61
Prof. Tobias Jahnke

Contents

When elliptic or parabolic differential equations on a domain with a non-trivial geometry have to be solved numerically, Finite Element Methods are often the first option. The central topic of this course is the mathematical analysis of this class of methods, in particular their construction, stability, and accuracy. The following aspects will be adressed:

Weak formulation of elliptic boundary value problems, well-posedness; Finite element methods for elliptic problems; Multigrid methods; Elliptic eigenvalue problems; Mixed methods for saddle point problems; Finite element methods for parabolic problems; Discontinuous Galerkin methods (if time permits).
This course is closely related to my lecture "Einführung in das Wissenschaftliche Rechnen" (summer term 2016), which was focused on practical aspects, in particular implementation. The current lecture, in contrast, will be devoted to theory, in particular well-posedness and error analysis. Both lectures are independent, i.e. "Finite Element Methods" can be attended without having attended "Einführung in das Wissenschaftliche Rechnen".
The course consists of a lecture and problem classes, both given in English.

Prerequisites

Participants are expected to have basics in numerical mathematics (methods for linear and nonlinear systems, numerical integration, interpolation etc., as taught in the courses "Numerische Mathematik 1+2"). Some knowledge in functional analysis (in particular Sobolev spaces) is helpful, but not a mandatory prerequisite. Programming skills are not required.

References

D. Braess: Finite Elemente. Theorie, schnelle Löser und Anwendungen in der Elastizitätstheorie. 5th revised ed., pp. xvi + 369, Springer Berlin, 2013.
S. C. Brenner and L. R. Scott: The mathematical theory of finite element methods. 3rd ed., pp. xvii + 397, Springer 2008.
M. Hanke-Bourgeois: Grundlagen der numerischen Mathematik und des wissenschaftlichen Rechnens, 3rd revised ed., Vieweg 2009.
P. Knabner, L. Angermann: Numerical methods for elliptic and parabolic partial differential equations. Translation from the German. pp. xv + 424, Springer 2003.

0104800 Functional Analysis

Lecture: 4 h, 8 credit points
Mon 9:45-11:15, 20.30, 1.067; Thu 9:45-11:15, 20.30, SR 1.067
Tutorial: 2 h
Fri 14:00-15:30, 20.40, Eiermann
Prof. Tobias Lamm

Contents

Functional Analysis uses concepts of the basic Linear Algebra courses such as vector space, linear operator, dual space, scalar product, adjoint map, eigenvalue, spectrum, in order to solve equations in infinite-dimensional function spaces, in particular linear differential equations.
The algebraic notations have to be extended by topological concepts such as convergence, completeness and compactness. This approach was only developed at the beginning of the 20th century, but nowadays it belongs to the methodological basis in Analysis, Numerics and Mathematical Physics, especially in Quantum Mechanics.

Prerequisites

Students attending this course should have attended the introductory lectures on Analysis and Linear Algebra. In particular, standard results on measure theory will be used without further explanation.

References

The course will be based on the following books:
Alt, H.W.: Linear Functional Analysis.
Brezis, H.: Functional Analysis, Sobolev Spaces and Partial Differential Equations.
Hirzebruch, F. ,Scharlau, W.: Einführung in die Funktionalanalysis.
Rudin, W.: Functional Analysis.

0109200 Numerical Methods for Maxwell's Equations

Lecture: 2 h, 6 credit points
Wed 9:45-11:15, 20.30, SR 3.61; Thu 9:45-11:15, 20.30, SR 3.61
(one of them will become the regular date for the lecture, the other an alternative date, this will be discussed in the first class on Wed, Oct 19)
Tutorial: 2 h
Mon 15:45-17:15, 20.30, SR 3.61
(this slot might also be used for the lecture, if necessary)
Prof. Marlis Hochbruck

Contents

Maxwell equations are a set of vector valued partial differential equations that are fundamental for the propagation of electromagnetic waves in media.
In this lecture we start to derive Maxwell equations in integral- and differential form, discuss examples of material laws, boundary conditions, and study the well-posedness in suitable function spaces.
For the numerical solution of Maxwell equations, we employ finite element methods for the spatial discretization. Our emphasis is on discontinuous Galerkin methods.
Favorable methods for time discretization are splitting methods, (locally) implicit schemes, and exponential integrators. We construct and analyse these methods and discuss their efficient implementation. Oral examination.

Prerequisites

The course is meant for advanced Master students who are familiar with the basics of finite element methods and numerical methods for differential equations. Some knowledge on functional analysis is also helpful.

References

D.A. Di Pietro, A. Ern, Mathematical Aspects of Discontinuous Galerkin Methods,
http://www.springer.com/de/book/9783642229794
A. Kirsch, A. Hettlich, The Mathematical Theory of Time-Harmonic Maxwell's Equations,
http://www.springer.com/de/book/9783319110851.

0150300 Combinatorics in the Plane

Lecture: 3 h, 7 credit points
Tue 11:30-13:00, 20.30, SR 3.61
Tutorial: 2 h
Wed 14:00-15:30, 20.30, SR 2.59
Dr. Torsten Ueckerdt

Contents

This course is an introduction to a variety of standard and non-standard concepts in plane combinatorics. This contains but is not limited to planar point sets, intersection patterns, order relations, and geometric arrangements. The concepts are presented in a problem-oriented form, i.e., each concept is motivated by a typical problem in the field, such as a coloring problem, an extremal question, a structural question, or a problem of representability.

Prerequisites

Students should know some basic concepts in discrete mathematics, such as binomial coefficients and graphs, as well as have some background in logic reasoning, such as the induction principle, double counting, and some linear algebra. The course is addressed to master students studying mathematics, computer science or a related subject.

References

Lecture notes are provided in pdf format. As supplementary material the following books are recommended:
Stefan Felsner. *Geometric Graphs and Arrangements.*
Jiří Matoušek. *Lectures on Discrete Geometry.*

0104600 Nonlinear Boundary Value Problems

Lecture: 4 h, 8 credit points
Tue 15:45-17:15, 20.30, SR 3.68; Fri 11:30-13:00, 20.30, SR 3.68
Tutorial: 2 h
Wed 15:45-17:15, 20.30, SR 3.68
Prof. Michael Plum

Contents

The lecture course will be concerned with boundary value problems for nonlinear elliptic partial differential equations, mainly of second order. In contrast to the linear case, no "unified" existence theory is at hand, but various approaches for proving existence (and other properties) of solutions need to be studied. The methods investigated in the lecture course are subdivided into non-variational and variational methods.
A preliminary and incomplete list of topics:
Motivating examples, monotonicity methods, fixed-point methods, super- and subsolutions, non-existence results, radial symmetry, a short introduction into variational calculus, Euler-Lagrange equations, variational problems under constraints, critical points, mountain pass theorem, perturbation results.

Prerequisites

Knowledge in functional analysis (Hilbert- and Banach spaces, weak convergence, dual space, Frechet differentiable operators) is essential, as well as the Lebesgue integral and Sobolev spaces. Knowledge in the classical theory of partial differential equations, and about weak solutions to linear problems, will be very useful.

References

Will be given in the first week of the semester.