

Seminar: Completely Integrable Systems I: Mathematical Methods of Classical Mechanics (SS21)

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Contents: We propose a series of seminars starting from Summer Semester 2021, each of which would be independent and self-contained. The ultimate goal is to study some interesting completely integrable Hamiltonian Partial Differential Equations (PDEs) which have *Lax-pair* formulations and are *completely integrable* by means of *inverse scattering methods*. Typical examples include the Korteweg-de Vries equation (KdV) and the one-dimensional cubic nonlinear Schrödinger equations (NLS).

To have a clear geometric picture of these, in this semester SS21 we shall start by studying mathematical methods in *classical mechanics*. Our plan is to go through the Lagrangian formalism of classical mechanics, while emphasizing on some concrete classical integrable systems such as central force problems and integrable rigid body motions. We shall follow the book “Mathematical Methods of Classical Mechanics” by V.I. Arnold [1] and we propose the following topics:

1. Experimental facts. (Sections 1-3 [1])
2. Systems with one/two degree(s) of freedom. (Sections 4-5 [1])
3. Central fields problems. (Sections 6-9 [1])
4. Conservation laws and similarity methods. (Sections 10-11 [1])
5. Variational principles. (Sections 12-14 [1])
6. Liouville's theorem. (Sections 15-16 [1])
7. Lagrangian dynamical systems. (Sections 17-19 [1])
8. Noether's theorem and D'Alembert's principle. (Sections 20-21 [1])
9. Linearization and small oscillations. (Sections 22-23 [1])
10. Characteristic frequencies and parametric resonance. (Sections 24-25 [1])
11. Inertial forces and Coriolis force. (Sections 26-27 [1])
12. Rigid bodies. (Sections 28-29 [1])
13. Lagrange's top. (Sections 30-31 [1])

In the next semester we plan to continue reading this book with the purpose to understand, among others, the Liouville-Arnold theorem on the existence of action-angle coordinates for a finite dimensional integrable Hamiltonian system.

Reference:

1. V.I. Arnold, *Mathematical Methods of Classical Mechanics*. Second Edition. Springer. 1989.

Prerequisites: Analysis I-II, Linear Algebra.

Remarks: This seminar will be held online. If you are interested in participating in this seminar, please write an email to xian.liao@kit.edu before 22.03.2021. There will be an online meeting to distribute the topics at the end of March.