Support measures and complex dimensions

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Abstract

We study new geometric functionals for arbitrary compact subsets of the $d$-dimensional Euclidean space called the basic and support functionals. The basic functionals are tightly related to the general Steiner-like formula of Hug, Last and Weil involving the support measures of the given set. By introducing appropriate critical scaling exponents of the basic functionals we connect them to the upper Minkowski dimension of the given set. On the other hand, a Steiner-like formula for arbitrary compact sets can also be derived by using the theory of complex fractal dimensions and the associated distance and tube zeta functions. In order to connect and complement these two theories, we introduce new (fractal) zeta functions corresponding to the basic and support functionals and provide functional equations connecting them to the classical fractal zeta functions. This provides a first glimpse on how the complex dimensions are generated by the basic functionals of the set. Furthermore, we also comment on the connection to the theory of fractal curvature measures. We provide interesting examples to support our results. This is a joint work with S. Winter.