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Applied Stochastic Models (SS 09)

Problem Set 1

Problem 0

Let X_1, $X_2 \sim U(\theta - 1/2, \theta + 1/2)$ be independent. Show that the probability density function (pdf) of $X_1 - X_2$ is independent of θ.

Problem 1

Let X and Y be random variables with distribution functions F and G respectively. We say that X is *stochastically dominated* or *dominated in distribution* by Y if

$$F(x) \geq G(x), \quad x \in \mathbb{R}.$$

Denote this relation by $X \overset{d}{\leq} Y$. Further, if F is a distribution function, its generalized inverse $F^{-1} : (0,1) \to \mathbb{R}$ is defined by

$$F^{-1}(u) = \inf\{x \in \mathbb{R} : F(x) \geq u\}, \quad u \in (0,1).$$

(a) Prove that $F^{-1}(u) \leq x$ if and only if $u \leq F(x)$.

(b) Prove that if $U \sim U[0,1]$, then $X \overset{d}{=} F^{-1}(U)$.

(c) Prove that $X \overset{d}{\leq} Y$ if and only if there exist two random variables $\hat{X}$ and $\hat{Y}$ such that $\hat{X} \overset{d}{=} X$ and $\hat{Y} \overset{d}{=} Y$ (i.e. two *copies* of X and Y) with the property $\hat{X} \leq \hat{Y}$.

Problem 2

Suppose X is distributed according to a Poisson distribution with random parameter Λ which is $\Gamma(a,c)-$distributed $(a > 0, c > 0)$. Assume in addition $c \in \mathbb{N}$. Find the distribution of X.

Problem 3

Let $X, Y \sim U(0,1)$ be independent. Write $U := \min\{X,Y\}$ and $V := \max\{X,Y\}$. Find $\mathbb{E}[U], \mathbb{E}[V]$ and calculate $\mathrm{Cov}(U,V)$.

Problem 4

Let $n \in \mathbb{N}$ and $Y \sim \mathrm{Bin}(n,X)$, where X is a random variable following a beta distribution on $(0,1)$ (with parameters $p, q > 0$). Find the distribution of Y. What happens if X is uniform on $(0,1)$?