

## Applied Stochastic Models (SS 09)

### Problem Set 4

#### Problem 1

Find the mean and variance of the Weibull distribution with hazard rate (or failure rate)

$$r(t) = \frac{f(t)}{1 - F(t)} = \lambda\alpha(\lambda t)^{\alpha-1}, \lambda > 0, \alpha > 0, t \geq 0.$$

#### Problem 2

Let  $N(t)$  be a Poisson process, and let  $Y_1, Y_2, Y_3, \dots$  be independent and identically distributed random variables. Find the mean and variance of

$$\sum_{i=1}^{N(t)} Y_i.$$

#### Problem 3

Consider a homogeneous Poisson process  $N(t)$  with random (and with respect to time constant) intensity  $\lambda$  which takes two values  $\lambda_1, \lambda_2$  with equal probabilities. (This means that first, someone rolls the intensity according to a fair coinflip, and then generates the Poisson process with this intensity.) Find the probability generating function of  $N(t)$ .

#### Problem 4

Let the times between the events of a renewal process  $N(t)$  be uniformly distributed on  $(0, 1)$ . Find the mean and variance of  $N(t)$  for  $0 < t < 1$ .