

Applied Stochastic Models (SS 09)

Problem Set 5

Problem 1

The r^{th} point T_r of a Poisson process $N(t)$ of constant intensity λ gives rise to an effect

$$X_r e^{-\alpha(t-T_r)}$$

at time $t \geq T_r$, where the X_r are independent and identically distributed with finite variance. Find the characteristic function (or Laplace transform) of the total effect

$$S(t) = \sum_{r=1}^{N(t)} X_r e^{-\alpha(t-T_r)}$$

and its first two moments in terms of the first two moments of the X_r , and calculate $\text{Cov}(S(s), S(t))$. Show that

$$\rho(S(s), S(s+v)) \rightarrow e^{-\alpha v} \text{ as } s \rightarrow \infty.$$

Problem 2

Let $N(t)$ be a non-homogeneous Poisson process with intensity function $\lambda(t)$. Show that the joint density function of the first two inter-event times is given by

$$\lambda(x)\lambda(x+y)e^{-\int_0^{x+y} \lambda(u)du}$$

and deduce that they are not in general independent.

Hint: Start with $P(T_1 \leq x, T_2 - T_1 > y)$.

Problem 3

Show that for a structure function ϕ

(a) if $\phi(0, 0, \dots, 0) = 0$, $\phi(1, 1, \dots, 1) = 1$, then $\min x_i \leq \phi(\mathbf{x}) \leq \max x_i$,

(b) $\phi(\max(\mathbf{x}, \mathbf{y})) \geq \max(\phi(\mathbf{x}), \phi(\mathbf{y}))$,

(c) $\phi(\min(\mathbf{x}, \mathbf{y})) \leq \min(\phi(\mathbf{x}), \phi(\mathbf{y}))$.

Problem 4

For any structure function ϕ , we define the dual structure ϕ^D by

$$\phi^D(\mathbf{x}) = 1 - \phi(\mathbf{1} - \mathbf{x}), \quad \text{where } \mathbf{1} := (1, 1, \dots, 1).$$

- (a) Show that the dual of a parallel (series) system is a series (parallel) system.
- (b) Show that the dual of the dual structure is the original structure.
- (c) What is the dual of a k -out-of- n structure?
- (d) Show that a minimal path (cut) set of the dual system is a minimal cut (path) set of the original structure.