University of Karlsruhe
Institute of Stochastics

Prof. Dr. P. R. Parthasarathy
Dipl.-Math. D. Gentner

Applied Stochastic Models (SS 09)

Problem Set 11

Problem 1

Describe three distinct methods to generate a random variable with density

$$f(x) = 6x(1-x), \quad 0 \le x \le 1.$$

Problem 2

Let X be a non-negative integer-valued random variable with

$$h(r) = P(X = r | X \ge r).$$

If $U_i, i = 1, 2, 3, ...$ are independent and uniform in $[0,1]$, show that $Z = \min\{n : U_n \le h(n)\}$ has the same distribution as X.

Problem 3

Suppose it is easy to simulate from the distributions $F_i, i = 1, 2, ..., n$. Give a procedure to simulate

$$F(x) = \sum_{i=1}^{n} p_i F_i(x), \quad p_i > 0, \sum p_i = 1.$$

Then give a method to simulate from

$$F(x) = \begin{cases} \frac{1-e^{-2x}+2x}{3}, & 0 < x < 1, \\ \frac{3-e^{-2x}}{3}, & 1 < x < \infty. \end{cases}$$

Problem 4

For a (non-homogeneous) Poisson process with intensity function $\lambda(t), t \ge 0$, where

$$\int_0^\infty \lambda(t) dt = \infty,$$

let $X_1, X_2, ...$ denote the sequence of times at which events occur. Show that

$$\int_{X_{i-1}}^{X_i} \lambda(t) dt, \quad i \ge 1,$$

are independent exponential with rate 1 where $X_0 = 0$.