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Applied Stochastic Models (SS 09)

Problem Set 12

Problem 1

(a) Two random variables X and Y are identically distributed. Show that

$$Var((X+Y)/2) \leq Var(X).$$

Conclude that the use of antithetic variables can never increase variance.

(b) If the random variables $(X_1, X_2, ..., X_n) =: X$ are independent and f and g are increasing functions of these n variables, use induction to prove that

$$\mathbb{E}(f(X)g(X)) \geq \mathbb{E}(f(X))\mathbb{E}(g(X)).$$

Problem 2

Generate 300 pairs of random numbers and use them to simulate an $M/\Gamma(2,2)/1$ queue. Arrivals are exponential with mean 2 and service times are gamma with parameters (2,2). Obtain the average waiting time of the customers.

Problem 3

A point process consisting of randomly occurring points in the plane is said to be a two-dimensional Poisson process having rate λ if the number of points in any given area A is Poisson distributed with mean $\lambda|A|$ ($|\cdot|$ denotes area or 2-dim Lebesgue measure) and the numbers of points in disjoint regions are independent.

Write an algorithm to simulate points of this process in a circular region of radius r centered around a fixed point O.

(Hint: Let $R_i, i = 1, 2, 3, ...$ denote the distance between O and its i^{th} nearest Poisson point. Then $P(\pi R_1^2 > b) = e^{-\lambda b}, P(\pi R_2^2 - \pi R_1^2 > b | R_1) = e^{-\lambda b}$, ...)

Problem 4

Use the Gibbs sampler to generate n random points in the unit circle conditional on the event that no two points are within a distance d of each other ($d < \frac{2\pi}{n}$), where

$$P(\text{no two points are within } d \text{ of each other})$$

is assumed to be a small positive number.