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Applied Stochastic Models (SS 09)

Problem Set 13

Problem 1

Control charts for $\overline{X}$ and R are to be set up for an important quality characteristic. The sample size is $n = 5$, and

$$\sum_{i=1}^{35} \overline{x_i} = 7805, \quad \sum_{i=1}^{35} R_i = 1200.$$

(a) Find trial control limits for $\overline{X}$ and R charts.
(b) Assuming that the process is in control, estimate the process mean and standard deviation.
($A_2 = 0.577$)

Solution: The grand sample mean is $\overline{\overline{x}} = \frac{7805}{35} = 223$ and the mean range is $\overline{r} = \frac{1200}{35}$. Per definition of the $\overline{X}$-chart the upper control limit is

$$UCL = \overline{\overline{x}} + A_2\overline{r} = 223 + 0.577 \cdot 34.286 = 242.78,$$

while the lower control limit is

$$LCL = \overline{\overline{x}} - A_2\overline{r} = 223 - 0.577 \cdot 34.286 = 203.22.$$

Per definition of the R-chart the UCL here is given by $D_4\overline{r} = 2.115 \cdot 34.286 = 72.51$ while the LCL is given by $D_3\overline{r} = 0 \cdot 34.286 = 0$.
(b) The empirical process mean is equal to $\overline{\overline{x}} = 223$ and the empirical standard deviation is just $\frac{\overline{r}}{d_2} = \frac{34.286}{2.326} = 14.74$.

Problem 2

A sampling plan, calling for a sample of size $n = 50$, has the acceptance number $c = 3$. Assuming that the lot size is very large, calculate the probability of accepting a lot of incoming quality 15% defective and rejecting a lot of incoming quality 4% defective
(a) by using the binomial probabilities and
(b) by using the Poisson approximation of the binomial distribution.
(c) In the above problem, use the Poisson approximation to calculate $L(p)$ for $p = 0.01, 0.02, ..., 0.20$. Sketch the OC curve for this sampling plan and read off the consumers and producers risk corresponding to an AQL of 4% and an LTPD of 14%.

Solution: (a) The probability of accepting a lot with incoming quality $p = 0.15$ by a sample size of 50 and acceptance number $c = 3$ is (assuming a large lot size, hence using the binomial approximation of the hypergeometric distribution) given by

$$\sum_{i=0}^{3} \binom{50}{i} p^i (1-p)^{50-i} = 0.046.$$

The probability of rejecting a lot with incoming quality $p = 0.04$ by a sample size of 50 and acceptance number $c = 3$ is given by

$$\sum_{i=4}^{50} \binom{50}{i} p^i (1-p)^{50-i} = 0.139.$$

(b) Using the Poisson Approximation of the binomial distribution, the above probabilities can be roughly obtained as follows: the parameter λ_1 is then approximately $np_1 = 7.5$, while $\lambda_2 = np_2 = 2$. The respective probabilities are

$$\sum_{i=0}^{3} e^{-\lambda_1} \frac{\lambda_1^i}{i!} = 0.059$$

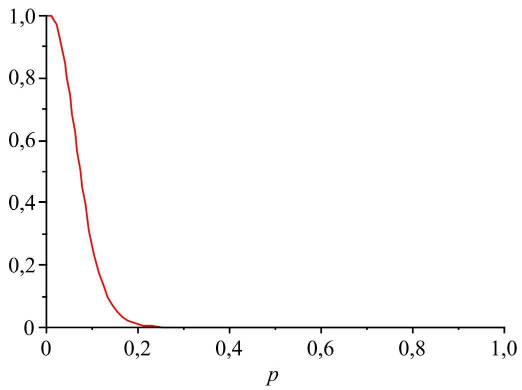
and

$$\sum_{i=4}^{50} e^{-\lambda_2} \frac{\lambda_2^i}{i!} = 0.142.$$

(c) Using the Poisson approximation we find

$$L(p) \simeq \sum_{i=0}^{3} e^{-np} \frac{(np)^i}{i!},$$

which yields the following curve:



If the AQL is 4% then the producers risk (or type one error) is $1 - L(0.04) = 0.143$ and if the LTPD is 14% then the consumers risk (or type two error) is $L(0.14) = 0.082$.

Problem 3

25 successive samples of 200 switches , each taken from a production line, contained, respectively,

$$6, 7, 13, 7, 0, 9, 4, 6, 0, 4, 5, 11, 6, 18, 4, 1, 9, 8, 2, 17, 9, 12, 10, 5, \text{ and 4 defectives.}$$

If the fraction defective is to be maintained at 0.02, construct a p chart for these data and state whether or not this standard is being met.

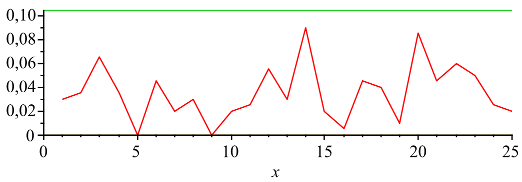
Solution: Since there is a intended standard of $p = 0.02$ given, the upper and lower 3-σ-control limits of our p-chart are given by

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}}$$

and

$$LCL = \max(p - 3\sqrt{\frac{p(1-p)}{n}}, 0).$$

The respective sample size is $n = 200$, which yields $UCL = 0.104$ and $LCL = 0$. The p-chart is the following diagram and it shows that the process is in control, hence the standard is being met.



Problem 4

Consider the following double sampling plan. The first sample has size 15 and the acceptance number is 1, the rejection number is 5 and the second sample has size 30, the acceptance number is 5 and the rejection number is 6. If the incoming quality is $p = 0.05$, determine the average sample number.

Solution: The random total sample number is given by

$$X(\omega) = 15 + 30\mathbb{1}_A(\omega),$$

where A is the event that a second sample is being drawn. We have

$$P(A) = P(\text{number of defectives is 2,3 or 4 in first sample of size 15})$$

which is equal to

$$\binom{15}{2}p^2(1-p)^{13} + \binom{15}{3}p^3(1-p)^{12} + \binom{15}{4}p^4(1-p)^{11} = 0.170,$$

and hence

$$\mathbb{E}X = 15 + 0.170 \cdot 30 = 20.1.$$