University of Karlsruhe
Institute of Stochastics

Prof. Dr. P. R. Parthasarathy
Dipl.-Math. D. Gentner

Applied Stochastic Models (SS 08)

Problem Set 2

Problem 1

Let X, Y, Z be independent exponential variables with respective parameters λ, μ, ν. Find

$$P(X < Y < Z).$$

Problem 2

Let (X, Y) have the 2-dimensional standard normal distribution with zero means, unit variances and correlation coefficient ρ. Write $Z := \max\{X, Y\}$. Show that $\mathbb{E}(Z) = \sqrt{(1-\rho)/\pi}$ and $\mathbb{E}(Z^2) = 1$.

Problem 3

Let $X \sim \Gamma(\lambda, \alpha)$ and $Y \sim \Gamma(\lambda, \beta)$ be independent. Show that $X + Y$ and $X/(X+Y) =: Z$ are independent and that Z follows a beta distribution with parameters α, β.

Problem 4

Let U be uniform on $[0, 1]$ and $0 < q < 1$. Show that $X := 1 + [\ln(U)/\ln(q)]$ has a geometric distribution.