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Applied Stochastic Models (SS 08)

Problem Set 5

Problem 1

The r^{th} point T_r of a Poisson process $N(t)$ of constant intensity λ gives rise to an effect

$$X_r e^{-\alpha(t-T_r)}$$

at time $t \geq T_r$, where the X_r are independent and identically distributed with finite variance. Find the characteristic function (or Laplace transform) of the total effect

$$S(t) = \sum_{r=1}^{N(t)} X_r e^{-\alpha(t-T_r)}$$

and its first two moments in terms of the first two moments of the X_r, and calculate $Cov(S(s), S(t))$. Show that

$$\rho(S(s), S(s+v)) \to e^{-\alpha v} \text{ as } s \to \infty.$$

Problem 2

Let $N(t)$ be a non-homogeneous Poisson process with intensity function $\lambda(t)$. Show that the joint density function of the first two inter-event times is given by

$$\lambda(x)\lambda(x+y)e^{-\int_0^{x+y}\lambda(u)du}$$

and deduce that they are not in general independent.
Hint: Start with $P(T_1 \leq x, T_2 - T_1 > y)$.

Problem 3

Show that for a structure function ϕ

(a) if $\phi(0,0,...,0) = 0, \phi(1,1,...,1) = 1$, then $\min\ x_i \leq \phi(\mathbf{x}) \leq \max\ x_i$,

(b) $\phi(\max\ (\mathbf{x},\mathbf{y})) \geq \max\ (\phi(\mathbf{x}), \phi(\mathbf{y}))$,

(c) $\phi(\min\ (\mathbf{x},\mathbf{y})) \leq \min\ (\phi(\mathbf{x}), \phi(\mathbf{y}))$.

Problem 4

For any structure function ϕ, we define the dual structure ϕ^D by

$$\phi^D(\mathbf{x}) = 1 - \phi(\mathbf{1} - \mathbf{x}), \quad \text{where } \mathbf{1} := (1,1,...,1).$$

(a) Show that the dual of a parallel (series) system is a series (parallel) system.

(b) Show that the dual of the dual structure is the original structure.

(c) What is the dual of a k-out-of-n struture?

(d) Show that a minimal path (cut) set of the dual system is a minimal cut (path) set of the original structure.