

Applied Stochastic Models (SS 08)

Problem Set 8

Problem 1

Consider a series system with two identical components with gamma distributed lifetimes with $\alpha = 5$ and arbitrary λ . Obtain the mean system lifetime.

Problem 2

Give an example to show that a k -out-of- n -system with independent and non-identical IFR lifetime components need not be IFR.

Problem 3

(a) It costs more to produce an item with a large expected life length than one with a small life expectancy. Suppose that the cost C of producing an item is given as $C = 3\mu^2$ where μ is the mean time to failure. Assume that a profit of D is realized for every hour the item is in service. Determine the maximum expected profit per item.

(b) Suppose there is a penalty cost K involved per unit time the unit fails before t_0 . Specifically, the random profit $P(T)$ per item in terms of the random lifetime T is given by

$$P(T) = \begin{cases} DT - 3\mu^2, & T > t_0, \\ DT - 3\mu^2 - K(t_0 - T), & T < t_0 \end{cases}.$$

Explain how the maximum expected profit can be found in this case assuming exponential lifetime T .

Problem 4

(a) Draw a sample of size n from Weibull distributed random variables and denote the smallest drawn value with $X_{(1)}$. Prove that $X_{(1)}$ also follows a Weibull distribution.

(b) Draw a sample of size n from random variables each with invertible cdf $F(t)$. Show that $n(1 - F(X_{(n)}))$ tends to an exponential distribution.

Solution: