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Applied Stochastic Models (SS 08)

Problem Set 9

Problem 1

Consider a system of two nonindependent components. Three types of shocks occur at times U_1, U_2, U_{12} following exponential variables with different parameters $\lambda_1, \lambda_2, \lambda_{12}$. Shock of type I destroys component 1, of type II destroys component 2 and of type III destroys both. Let $X = \min(U_1, U_{12})$ and $Y = \min(U_2, U_{12})$. Show that

(a) $P(X > x, Y > y) = e^{-(\lambda_1 x + \lambda_2 y + \lambda_{12} \max(x,y))}$,
(b) $P(\min(X, Y) > t) = e^{-(\lambda_1 + \lambda_2 + \lambda_{12})t}$,
(c) $P(X > x + t, Y > y + t \mid X > t, Y > t) = P(X > x, Y > y)$.
(d) Find the mean and variance of X.

Problem 2

An aircraft has four engines, each of which has a failure rate λ. For a successful flight at least two engines should be operating.

(a) Find the reliability $R(t)$ and expected lifetime of the aircraft.
(b) Find these if the aircraft needs at least one operating engine on either side for a successful flight.

Problem 3

Prove:

(a) If $0 \leq \alpha, \lambda \leq 1$, then

$$h(y) = \lambda^\alpha + (1 - \lambda^\alpha)y^\alpha - (\lambda x + (1 - \lambda)y)^\alpha \geq 0.$$

(*Hint*: Note that $f(t) = t^\alpha$ is a concave function, so that $f(t + h) - f(t)$ is decreasing in t.)
(b) Deduce that $r(\mathbf{p}^\alpha) \geq [r(\mathbf{p})]^\alpha, 0 \leq \alpha \leq 1$.

Problem 4

We say that ζ is a p-quantile of the distribution F if $F(\zeta) = p$. Show that if ζ is a p-quantile of the IFRA distribution F, then

$$\bar{F}(x) \leq e^{-\theta x}, \quad x \geq \zeta,$$

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where $\theta = \frac{-\ln(1-p)}{\zeta}$.