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# Applied Stochastic Models (SS 08)

Problem Set 12

**Problem 1**

(a) Two random variables $X$ and $Y$ are identically distributed. Show that

$$Var((X+Y)/2) \leq Var(X).$$

Conclude that the use of antithetic variables can never increase variance.
(b) If the random variables $(X_1, X_2, ..., X_n) =: X$ are independent and $f$ and $g$ are increasing functions of these $n$ variables, use induction to prove that

$$\mathbb{E}(f(X)g(X)) \geq \mathbb{E}(f(X))\mathbb{E}(g(X)).$$

**Problem 2**

Generate 300 pairs of random numbers and use them to simulate an $M/\Gamma(2,2)/1$ queue. Arrivals are exponential with mean 2 and service times are gamma with parameters (2,2). Obtain the average waiting time of the customers.

**Problem 3**

A point process consisting of randomly occurring points in the plane is said to be a two-dimensional Poisson process having rate $\lambda$ if the number of points in any given area $A$ is Poisson distributed with mean $\lambda|A|$ ($|\cdot|$ denotes area or 2-dim Lebesgue measure) and the numbers of points in disjoint regions are independent.
Write an algorithm to simulate points of this process in a circular region of radius $r$ centered around a fixed point $O$.
(Hint: Let $R_i, i = 1, 2, 3, ...$ denote the distance between $O$ and its $i^{th}$ nearest Poisson point. Then $P(\pi R_1^2 > b) = e^{-\lambda b}, P(\pi R_2^2 - \pi R_1^2 > b | R_1) = e^{-\lambda b}$, ... )

**Problem 4**

Use the Gibbs sampler to generate $n$ random points in the unit circle conditional on the event that no two points are within a distance $d$ of each other ($d < \frac{2\pi}{n}$), where

$$P(\text{no two points are within } d \text{ of each other})$$

is assumed to be a small positive number.