Model 1 Classic EOQ model

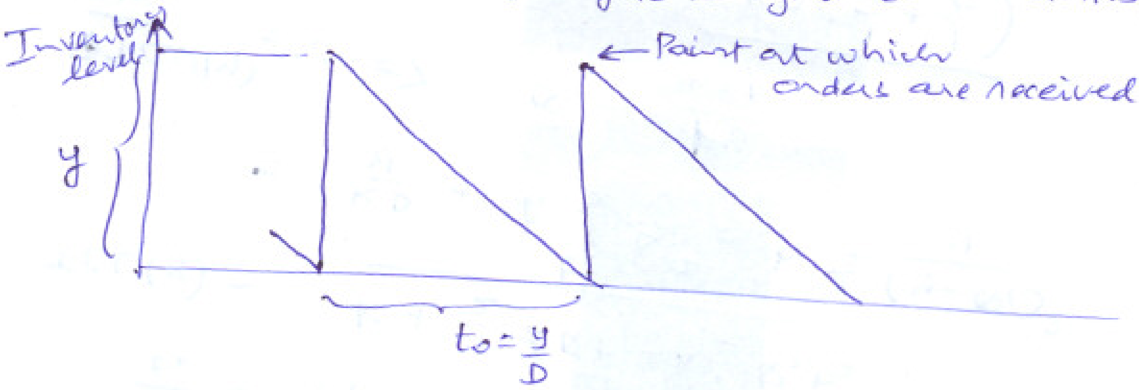
Constant-rate demand with instantaneous order replenishment and no shortage.

Define

y = order quantity (number of units)

D = Demand rate (units per unit time)

t_0 = Ordering cycle length (time units)



An order of size y units is placed and received instantaneously when the inventory level reaches zero level. The stock is depleted uniformly at rate D.

$$t_0 = \frac{y}{D} \text{ time units}$$

K = Setup cost associated with the placement of an order (€/order)

h = Holding cost (€/unit/unit time)

$TCU(y)$ = Total cost per unit time (TCU)

$$= \frac{\text{Setup cost} + \text{Holding cost per cycle } t_0}{t_0}$$

$$= \frac{K + h\frac{y}{2}t_0}{t_0}$$

$$= \frac{KD}{y} + h\frac{y}{2}$$

$$\frac{dTCU(y)}{dy} = -\frac{KD}{y^2} + \frac{h}{2} = 0$$

$TCU(y)$ is convex.

$$\therefore y^* = \sqrt{\frac{2KD}{h}}$$

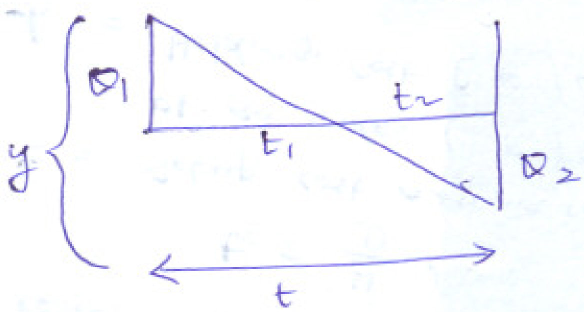
Minimum cost $= \sqrt{2KDh}$

$$t_0 = \frac{y^*}{D} = \sqrt{\frac{2K}{Dh}}$$

A positive lead time L occurs between the placement and the receipt of an order. The reorder point occurs when the inventory level drops to LD units.

Model 2 EOQ with shortage

Backlogging is permitted



$$y = Q_1 + Q_2, \qquad t = \frac{y}{D}$$

$$\frac{Q_2}{t_2} = \frac{Q_2}{t_1} = \frac{y}{t} = D$$

$$TCU(y) = \frac{K + \frac{1}{2} h Q_1 t_1 + p \frac{1}{2} Q_2 t_2}{t}$$

$$= \frac{KD}{y} + h \frac{1}{2} \frac{Q_1^2}{y} + p \frac{1}{2} \frac{(y-Q_1)^2}{y}$$

$$\frac{\partial TCU(y)}{\partial Q_1} = 0 \Rightarrow \quad Q_1 = \frac{py}{h+p}$$

$$\frac{\partial TCU(y)}{\partial y} = 0 \Rightarrow \quad y^* = \sqrt{2KD\left(\frac{1}{h}+\frac{1}{p}\right)}$$

(after substituting for Q_1)

$$\Rightarrow \quad Q^* = \sqrt{2KD \frac{ph}{p+h}}$$

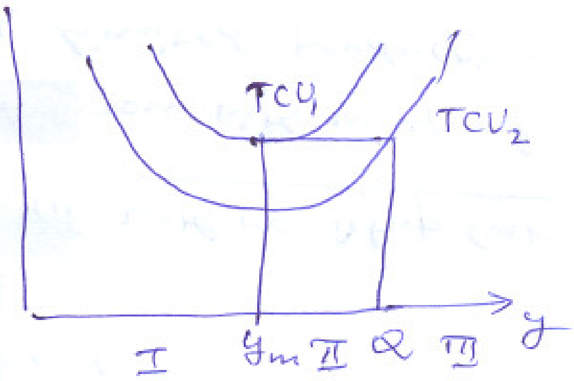
When $p = \infty$, it reduces to previous model

Model 3 — EOQ with Price Break

An Inventory item may be purchased at a discount if the size of an order exceeds the given limit Q. Let $c_1 > c_2$.

$$c = \begin{cases} c_1 & \text{if } y \le q \\ c_2 & \text{if } y > q. \end{cases}$$

$$TCU(y) = \begin{cases} Dc_1 + \frac{KD}{y} + \frac{h}{2}y, & y \le q \\ Dc_2 + \frac{KD}{y} + \frac{h}{2}y, & y > q \end{cases}$$



Without discount, $y_m = \sqrt{\frac{2KD}{h}}$

Divide y axis into three

$y < y_m$, $y_m < y < Q$, $y > Q$

From the figure,
order for y_m if q is in zone 1 or 3
and order q if q is in zone 2.

Model 4 — Multi item EOQ with storage limitation

There are n items which compete for a limited storage space. For $i = 1, 2, \ldots, n$, let

D_i = demand rate
K_i = Setup cost
h_i = holding cost / unit / unit time
y_i = order quantity
a_i = storage area / unit
A = Maximum available storage

We wish to

$$\text{Min } TCU(y_1, y_2, \ldots, y_m) = \sum_{i=1}^{m}\left(\frac{K_i D_i}{y_i} + \frac{h_i y_i}{2}\right)$$

such that $a_1 y_1 + \cdots + a_m y_m \le A$.

First compute $y_i^* = \sqrt{\frac{2K_i D_i}{h_i}}$. If y_i^* satisfies the constraint, these are the optimum levels, otherwise, we use Lagrangian multipliers.

i.e.,

$$\text{Min } L(\lambda, y_1, \ldots, y_m) = TCU(y_1, \ldots y_n) - \lambda\left(\sum a_i y_i - A\right)$$

The optimum values of y_i and λ are determined from $\frac{\partial L}{\partial y_i} = 0$, $\frac{\partial L}{\partial \lambda} = 0$.

$$-\frac{K_i D_i}{y_i^2} + \frac{h_i}{2} - \lambda a_i = 0 \Rightarrow y_i^* = \sqrt{\frac{2K_i D_i}{h_i - 2\lambda^* a_i}}$$

$$\sum a_i y_i = A$$

This being a minimization problem, $\lambda < 0$.

Successively decrease λ, ~~and~~ calculate y_i^* till ~~A~~ $\sum a_i y_i \simeq A$.

The above procedure can also be used when there is a limitation on investment.

Model 5 Dynamic EOQ model with no setup cost

There is a planning horizon with n equal periods. Each period has a limited production capacity that can include several production levels (e.g. regular time and overtime represent two production levels). ~~A~~

A current period may produce more than its immediate demand to satisfy demand for later periods, in which case an inventory holding cost must be charged.

The n-period problem can be formulated as a transportation model with kn sources and n destinations, where k is the number of production levels per period. The production capacity of each of the kn production-level sources provides the supply amounts. The demand amounts are specified by each demand's demand. The unit 'transportation' cost is the sum of applicable production and holding costs per unit. The solution of the problem as a transportation model determines the minimum-cost production in each production level.

<u>Model 6</u> <u>Dynamic EOQ model with setup cost</u>

For $i = 1, 2, \ldots, n$,

z_i = Amount ordered

D_i = Demand for period i

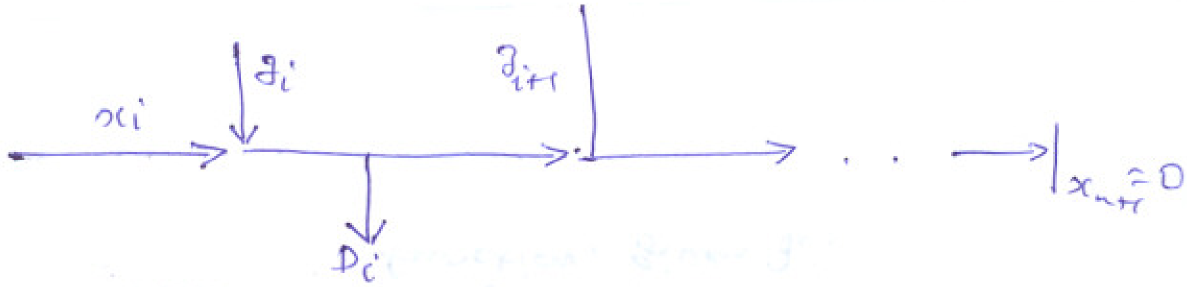
x_i = Inventory at the start of period i.

K_i = Setup cost.

h_i = Unit Inventory holding cost from period i to $i+1$

$$C_i(z_i) = \begin{cases} 0 & z_i = 0 \\ K_i + c_i(z_i) & z_i > 0 \end{cases}$$ = Production cost function for period i

$c_i(z_i)$ = Marginal production cost function, given z_i.



$$x_{i+1} = x_i + z_i - D_i$$

$$0 \leq x_{i+1} \leq D_{i+1} + \cdots + D_n$$

$$f_1(x_2) = \min_{z_1 = D_1 + x_2 - x_1} \left\{ C_1(z_1) + h_1(x_2) \right\}$$

$$f_i(x_{i+1}) = \min_{0 \leq z_i \leq D_i + x_{i+1}} \left\{ C_i(z_i) + h_i(x_{i+1}) + f_{i-1}(x_{i+1} + D_i - z_i) \right\}$$

$$i = 2, 3, \ldots, n$$

$$z_1 = D_1 + x_2 - x_1.$$