Probabilistic Inventory Models

We assume that the demand is random with a specified probability distribution.

Model 1

"Probabilitized" EOQ model

L = Lead time between placing and receiving an order

x_L = Demand during lead time

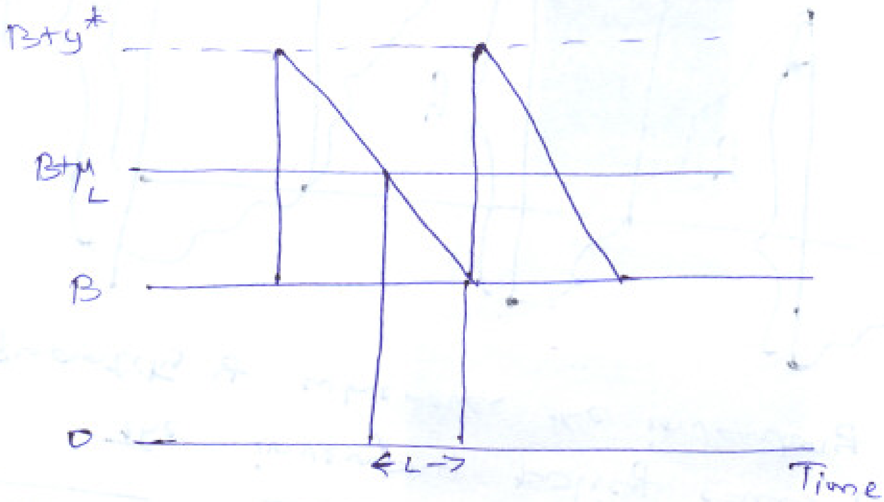
μ_L = $E(x_L)$

σ_L = S. D of x_L

B = Buffer stock size

α = Maximum allowable probability of running out of stock during lead time.

x_L is $N(\mu_L, \sigma_L^2)$.



$$P(x_L \geq B + \mu_L) \leq \alpha$$

$$P\left(\frac{x_L - \mu_L}{\sigma_L} \geq \frac{B}{\sigma_L}\right) \leq \alpha$$

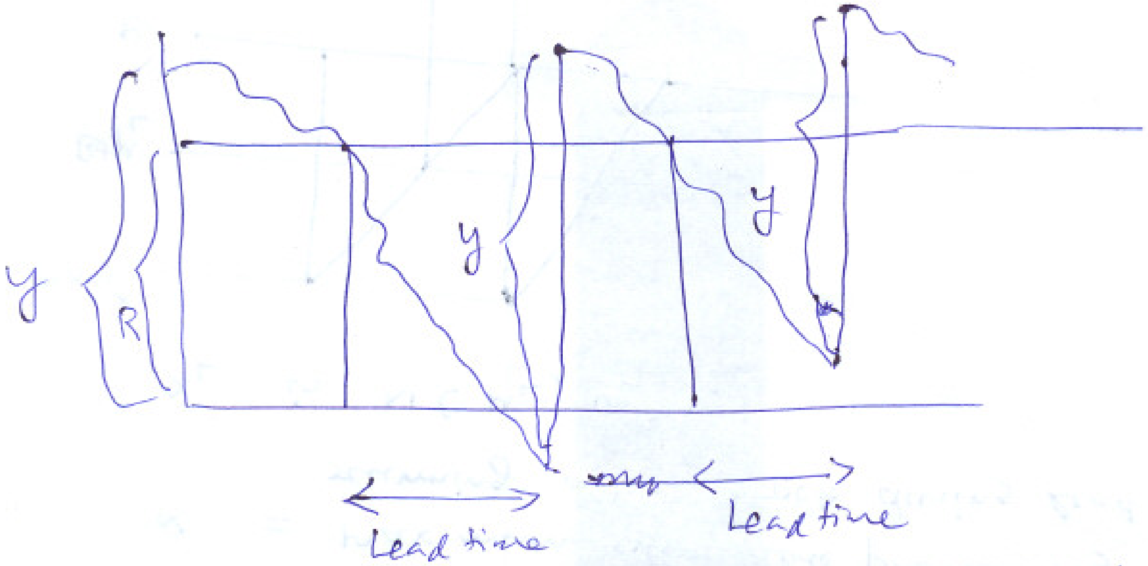
$$B \geq \sigma_L K_\alpha \quad (K_\alpha \text{ from normal tables})$$

Usually the demand during the lead time L is described by a pdf per unit time (per day or per week). Then

$$\mu_L = DL, \quad \sigma_L = \sqrt{\sigma^2 L}$$

Model 2 Probabilistic EOQ model

The inventory policy calls for ordering the quantity y whenever the inventory drops to level R.



The optimum values of y and R are determined by minimizing the expected cost per unit time = setup + holding + shortage cost.

We assume:

(1) Unfilled demand during lead time is backlogged

(2) No more than one outstanding order is allowed

$f(x)$ = pdf of demand, x, during lead time

D = Expected demand per unit time

h = holding cost / unit / unit time

p = shortage cost / unit

K = setup cost per order

~~Ste~~ Setup cost / unit time $= \frac{KD}{y}$

Expected holding cost $= Ih$ where

I = average inventory $= \frac{y + E\{R-x\} + E(R-x)}{2}$

$= \frac{y}{2} + R - E(x)$

$R \nless E(x)$

Expected shortage cost $= pS \cancel{*} \; \cancel{\int_R^\infty (x-R) f(x)\,dx}$

where $S = \int_R^\infty (x-R) f(x)\,dx$

is the shortage quantity per cycle.

~~TCU(y~~ Shortage cost / unit time $= \frac{pSD}{y}$

Total cost function per unit time is

$$TCU(y,R) = \frac{DK}{y} + h\left(\frac{y}{2} + R - E(x)\right) + \frac{pD}{y}\int_R^\infty (x-R) f(x)\,dx$$

$$\frac{\partial TCU(y)}{\partial y} = 0 \Rightarrow -\frac{DK}{y^2} + \frac{h}{2} - \frac{pDS}{y^2} = 0$$

$$\frac{\partial TCU(y)}{\partial R} = 0 \Rightarrow h - \frac{pD}{y}\int_R^\infty f(x)\,dx = 0$$

Thus, $y^* = \sqrt{\frac{2D(K+pS)}{h}}$, $\int_{R^*}^\infty f(x)\,dx = \frac{hy^*}{pD}$

For $R=0$, $\hat{y} = \sqrt{\frac{2D(K+pE(x))}{h}}$, $\tilde{y} = \frac{pD}{h}$

If $\tilde{y} \geq \hat{y}$, unique optimal values of y and R exist.

Algorithm

Step 0 $y_1 = y^* = \sqrt{\frac{2KD}{h}}$, $R_0 = 0$, set $i=1$, and go to step i

Step i use y_i to determine R_i ~~from (1)~~

$$y^* = \sqrt{\frac{2D(K+pS)}{h}}$$

If $R_i \simeq R_{i-1}$, stop. Then $y^* = y_i$, $R^* = R_i$

Otherwise, use R_i in

$$\int_{R^*}^{\infty} f(x)\,dx = \frac{hy^*}{pD}$$

to compute y_i. Set $i = i+1$, and repeat step i.

Model 3 Single-period with no setup cost

D = random demand during the period

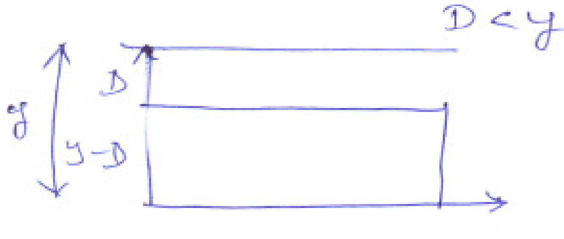
h = holding cost / unit during the period

p = penalty cost / unit during the period

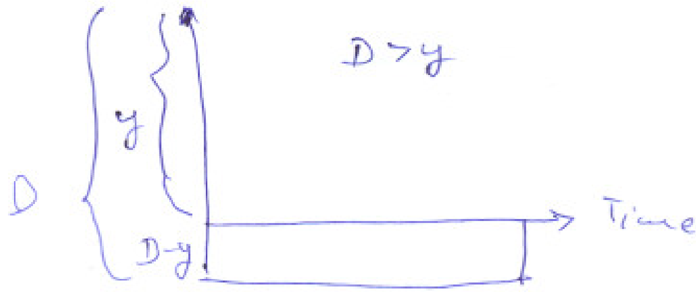
$f(D)$ = pdf

y = order quantity

x = Inventory on hand before an order is placed.



(a)



(b)

$$E\{C(y)\} = h\int_0^y (y-D)f(D)\,dD + p\int_y^\infty (D-y)f(D)\,dD$$

$$\frac{\partial E\{C(y)\}}{\partial y} = 0 \Rightarrow h\int_0^y f(D)\,dD - p\int_y^\infty f(D)\,dD = 0$$

$$P(D \le y^*) = \frac{p}{p+h}$$

If D is discrete,

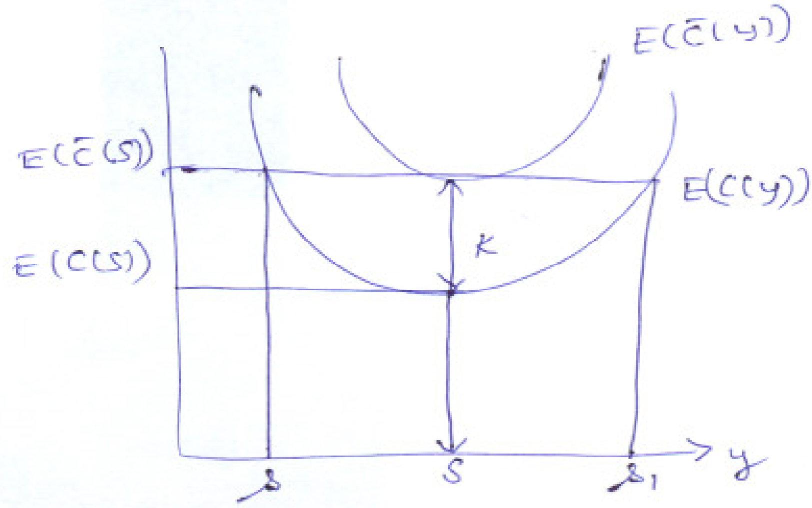
$$P(D \le y^*-1) \le \frac{p}{p+h} \le P(D \le y^*)$$

Model 4 Single period model with setup cost (s-S policy)

$$E(\bar{C}(y)) = K + E(C(y))$$

$$= K + h\int_0^y (y-D)f(D)\,dD + p\int_y^\infty (D-y)f(D)\,dD$$

$$P(y \le y^*) = \frac{p}{p+h}$$



~~Find~~ Let $S = y^*$.

Find the value of $s < S$ such that $E(C(s)) = K + E(C(S))$, $s < S$.

If $x < s$, order $S - x$.

If $x \ge s$ do not order.

(Take to Model 2)

If the demand is uniform in $(0, t)$

$$R^* = t\left(1 - \frac{h Q^*}{pD}\right)$$

$$Q^* = \sqrt{\frac{2DK + Dpt + Dp\frac{R^2}{t} - 2apR^*}{h}}$$