University of Karlsruhe
Institute of Stochastics

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Stochastic Methods in Industry I (WS 07/08)

Problem Set 1

Problem 1

A jar contains N chips with labels $1, ..., N$. We draw chips one by one **with** replacement. Let X denote the number of trials needed to draw a number which has been drawn already. Find $\mathbb{P}(X = k)$.

Hint: Which values of k are non-trivial? Find a suitable sample space Ω.

Problem 2

Let X be a random variable with probability density

$$f(x) = \frac{1}{\pi\sqrt{1-x^2}} \quad , -1 < x < 1,$$

and Y another random variable with probability density

$$g(y) = ye^{-y^2/2} \quad , y > 0.$$

Assuming X and Y are independent, prove that $XY \sim \mathcal{N}(0, 1)$.

Hint: Derive a general formula for the density of XY using the transformation formula for probability densities with a suitable transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$.

Problem 3

Let X_1 and X_2 be independent exponentially distributed random variables and set

$$N = \mathbb{1}_{\{X_1 < X_2\}} + 2 \cdot \mathbb{1}_{\{X_1 > X_2\}}$$

and

$$U = \min\{X_1, X_2\}.$$

Prove that U and N are independent.

Problem 4

Let $X_1, X_2, ...$ be independent random variables with

$$\mathbb{P}(X_i = 1) = \mathbb{P}(X_i = -1) = \frac{1}{2} \quad , i \in \mathbb{N}.$$

Show that

$$\sqrt{\frac{3}{n^3}} \sum_{k=1}^{n} kX_k \xrightarrow{\mathcal{D}} \mathcal{N}(0, 1), \quad n \to \infty.$$

Hint: The Central Limit Theorem (either Lindeberg's version or Lyapunov's version) will help.

Due date Friday, November 2nd 2007 before class. (Sheets can be turned in right before class.) Please put your **name** and **student id number** on each sheet you turn in.