

Stochastic Methods in Industry I (WS 07/08)

Problem Set 2

Problem 1

A fair die is rolled repeatedly. Which of the following are Markov chains? For those that are, supply the transition probability matrix.

- (a) The largest number X_n shown up to the n^{th} roll.
- (b) The number W_n of sixes up to the n^{th} roll.
- (c) At time n , the time A_n since the most recent six.
- (d) At time n , the time B_n until the next six.

Problem 2

Let

$$P = \begin{pmatrix} 1-a & a \\ b & 1-b \end{pmatrix}, \quad 0 < a, b < 1.$$

Find

- (a) P^n ,
- (b) $\lim_{n \rightarrow \infty} P^n$.

Problem 3

Consider a time-homogeneous random walk X_n on \mathbb{Z} such that

$$P_{i(i+1)} = \mathbb{P}(X_{n+1} = i+1 | X_n = i) = p, \quad i \in \mathbb{Z}, n \in \mathbb{N},$$

and

$$P_{i(i-1)} = \mathbb{P}(X_{n+1} = i-1 | X_n = i) = q, \quad i \in \mathbb{Z}, n \in \mathbb{N}.$$

where $0 < p < 1, p + q = 1$. Determine P_{00}^n .

Problem 4

Consider two urns A and B containing a total of N balls. An experiment is performed in which at time n ($n = 1, 2, \dots$) a ball is selected at random from all N balls. The selected ball will then be put into urn A with probability p or into urn B with probability q ($p + q = 1$). The state of the system at time n can be characterized by the number X_n of balls in urn A . Determine the transition matrix for the Markov chain X_n .

Due date Friday, November 9th 2007 before class. (Sheets can be turned in right before class.) Please put your **name** and **student id number** on each sheet you turn in.