

Stochastic Methods in Industry I (WS 07/08)

Problem Set 3

Problem 1

Customers arrive at a bank according to a homogeneous Poisson process with rate λ . Suppose two customers arrived during the first hour. What is the probability that

- (a) both arrived during the first 20 minutes?
- (b) at least one arrived during the first 20 minutes?

Problem 2

For a (possibly non-homogeneous) Poisson process $\{X(t)\}$ with parameter $\lambda(t)$ find

$$\mathbb{P}(\{\text{number of events occurring in } (t_1, t_2) = n\}), \quad n \in \mathbb{N}.$$

If $\lambda(t) = t^2 + 2t, t \geq 0$, find the probability that n events occur between $t = 4$ and $t = 5$.

Problem 3

For a homogeneous Poisson process $\{X(t)\}$ with parameter λ , prove that

$$\text{Cov}(X(s), X(t)) = \lambda \min(s, t), \quad s, t \geq 0.$$

Problem 4

Consider two independent homogeneous Poisson processes $X(t)$ and $Y(t)$ with rates λ and μ respectively. Find the probability distribution of the number of events of Y between two successive events of X .

Due date Friday, November 16th 2007 before class. (Sheets can be turned in right before class.) Please put your **name** and **student id number** on each sheet you turn in and staple the sheets.