University of Karlsruhe
Institute of Stochastics

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Stochastic Methods in Industry I (WS 07/08)

Problem Set 4

Problem 1

Insurance claims are made at times distributed according to a Poisson process with rate λ. The successive amounts are iid with mean μ and are independent of the claim arrival times. Let S_i denote the time and C_i denote the amount of the i-th claim. The total discounted cost of all claims made up to time t equals

$$D(t) = \sum_{i=1}^{N(t)} e^{-\alpha S_i} C_i.$$

Here α is the discount rate and $N(t)$ the number of claims made up to time t. Find the mean of $D(t)$.

Problem 2

Mr. Müller runs a hot dog stall that opens at 8 AM. Arrivals occur according to a non-homogeneous Poisson process with time-dependent rate $\lambda(t)$, where

$$\lambda(t) = \begin{cases} 5 + 5t, & 0 \le t \le 3, \\ 20, & 3 \le t \le 5, \\ 20 - 2(t-5), & 5 \le t \le 9, \end{cases}$$

taking 8 AM as origin. Find the probability that no customer arrives between 8:30 AM and 8:40 AM and the expected number of arrivals in this period.

Problem 3

Consider a Poisson process $\{N(t)\}$ with parameter λ, where λ follows a gamma density with mean m/α and variance m/α^2. Find $P(N(t) = n)$, mean and variance of $N(t)$.

Problem 4

For a birth and death process, find the average time it takes to enter state $n+1$, starting from state n.

Due date Friday, November 23th 2007, 14:00 o'clock. This week, since the session next Friday needs to be canceled, the solutions must be turned in by 14:00 o'clock in front of the office of the Institute of Stochastics, room number 231, where a box is set up labeled "Übungsblätter: Stochastic Methods in Industry". Please put your **name** and **student id number** on each sheet you turn in and staple the sheets.