

## Stochastic Methods in Industry I (WS 07/08)

### Problem Set 5

#### Problem 1

Customers arrive at a railway station in accordance with a homogenous Poisson process with rate  $\lambda$ . If the train departs at time  $t$ , compute the mean of the sum of the waiting times of all customers arriving in  $(0, t)$ .

#### Problem 2

An urn initially contains one white and one black ball. At each stage, a ball is drawn and then replaced along with another ball of the same colour. Let  $\{Z_n\}$  denote the fraction of white balls in the urn after the  $n$ th replication. Show that  $\{Z_n : n \geq 1\}$  is a martingale.

#### Problem 3

Consider a birth and death process with birth and death parameters  $\lambda_n, n \geq 1$ , and  $\mu_n, n \geq 1$ , where  $\lambda_0 = 0$  so that 0 is an absorbing state. Find the probability of absorption into state 0 from a fixed initial state  $m \in \mathbb{N}$  and the respective mean time to absorption.

#### Problem 4

Consider a birth process  $\{X(t)\}$  with birth rate  $\lambda$  (i.e.  $\lambda_n = n\lambda, n \geq 1$ ) and  $X(0) = 1$ . Show that  $X(t)$  follows a geometric distribution. Find the distribution function of  $X_t(x)$ , the number of members at time  $t$  of age less than or equal to  $x$ . (Hint: condition on the value of  $X(t-x)$ ).

**Due date** Friday, November 30th 2007, 14:00 o'clock. Sheets can be turned in right before class. Please put your **name** and **student id number** on each sheet you turn in and staple the sheets.