University of Karlsruhe
Institute of Stochastics

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Stochastic Methods in Industry I (WS 07/08)

Problem Set 6

Problem 1

A matrix with nonnegative elements is said to be doubly stochastic if all rowsums are 1 and also all columnsums are 1. Prove that if an irreducible Markov chain with finite state space $\{1, 2, ..., N\}$ has a doubly stochastic transition probability matrix P, then a stationary distribution $\pi = (p_1, ..., p_N)$ must be the uniform distribution on $\{1, ..., N\}$.

Problem 2

Suppose that customers are in line to receive service that is provided sequentially by a server. Whenever a service is completed, the next person in line enters the service facility. The service times are independent exponentially distributed with parameter μ. Each waiting customer will only wait an exponentially distributed time with rate Θ; if the service has not yet begun by this time he will immediately depart the system. Suppose that someone is presently being served and consider the person who is n^{th} in line. Find the probability that this customer will be eventually served and the conditional expected amount of time this person waits in line, given that he is eventually served.

Problem 3

A single repairman looks after two machines 1 and 2. Each time machine i is repaired, it stays up for an exponential time with rate λ_i, $i = 1, 2$. When machine i breaks down, it requires an exponentially distributed amount of work with rate μ_i to complete the repair. The repairman will always prioritize service machine 1 when it is down. For instance, if machine 1 breaks down while 2 is being repaired , then the repairman will immediately stop work on machine 2 and start on 1. What proportion of time is machine 2 down?

Problem 4

Consider a variant of an $M/M/2$ queue where the service rates μ_1, μ_2 of the two processors are not identical. Assume $\mu_1 > \mu_2$. The state of the system can be characterized by the pair (n_1, n_2), where $n_1 \geq 0$ denotes the number of jobs in the queue including the person at the fast server and $n_2 \in \{0, 1\}$ says, whether a person is currently served by the other server. Jobs wait in line in the order of the arrival. When both servers are idle, the faster server is scheduled for service before the slower one. Assume that the arrival rate of jobs is λ. Find the condition for stationarity to exist. Write the balance equations in steady state, the steady state probabilities and the mean number of jobs in the system.

Due date Friday, December 7th 2007, 14:00 o'clock. Sheets can be turned in right before class. Please put your **name** and **student id number** on each sheet you turn in and staple the sheets.