University of Karlsruhe
Institute of Stochastics

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Stochastic Methods in Industry I (WS 07/08)

Problem Set 8

Problem 1

In a county hospital, the time a doctor spends with a patient tends to decrease as the number of patients waiting increases. The average time a doctor spends with a patient is 24 minutes if no other patients are waiting, whereas this average becomes 12 minutes if 5 patients are waiting (i.e. if there are 6 patients in total in the system). Arrival of patients is according to a Poisson process with rate 2 per hour. Use the density-dependent service model (model 11), i.e. $\lambda_n = 2, n = 0, 1, 2, ...$, $\mu_n = n^\alpha \mu_1, n = 1, 2, ...$ (α suitable), to find the average time spent in the hospital by a patient if
(a) there is one doctor
(b) there are two doctors
correct up to 3 decimal places (Answers are 0.636 and 0.417 hours).

Problem 2

In an impatient customer system (i.e. an $M/M/c$ queue in which the customers will only wait an exponentially distributed amount of time with parameter ν to get service and immediatly leave the system if service has not begun by this time), assume $c = 1, \lambda = 2, \mu = 5$ and $\nu = 3$. Find p_0 correct up to 3 decimals.

Problem 3

For a state dependent finite queueing system with parameters $\lambda_{n-1}, \mu_n, n = 1, 2, 3, ...N$ obtain the steady state probabilities. Follow the method outlined in class, i.e. write down the Kolmogorov equations, take Laplace transforms, apply Cramers rule to the resulting linear system of equations, assume that the denominator has distinct roots $0 = s_0 > s_1 > s_2 > ... > s_N$, split in partial fractions, apply inverse Laplace transform, examine the limit for $t \to \infty$ and finally prove that for the steady state probabilities

$$p_n = \frac{\frac{\lambda_0 ... \lambda_{n-1}}{\mu_1 ... \mu_n}}{1 + \frac{\lambda_0}{\mu_1} + \frac{\lambda_0 \lambda_1}{\mu_1 \mu_2} + ... + \frac{\lambda_0 ... \lambda_{N-1}}{\mu_1 ... \mu_N}}.$$

Problem 4

Find the time-dependent probabilities $P_n(t)$ of
(a) an $M/M/1/1$ system,
(b) an $M/M/1/2$ system,
(c) an $M/M/2/2$ system.

Due date Friday, December 21st 2007, 14:00 o'clock. Sheets can be turned in right before class. Please put your **name** and **student id number** on each sheet you turn in and staple the sheets.