University of Karlsruhe
Institute of Stochastics

Prof. Dr. P. R. Parthasarathy
Dipl.-Math. D. Gentner

Stochastic Methods in Industry I (WS 07/08)

Problem Set 9

Problem 1

Consider a two-server queueing system where server i has rate μ_i for $i = 1, 2$. Arrivals to the system follow a homogeneous Poisson process of rate λ. Whenever the first server is free an arriving customer will always go to the first server. A customer arriving when the first server is occupied goes to the second server. If both servers are occupied, the customer is lost. Define the states of the system and find the stationary probabilities.
Then suppose that the arrival rate is 10 per hour. One of the μ_i is 6 per hour, the other shall be 4 per hour. Recommend which one should be placed first.

Problem 2

The two tellers in a bank take an exponentially distributed time to deal with any customer; their parameters are λ and μ respectively. You arrive to find exactly two customers present, each occupying a teller.
(a) You flip a fair coin to determine the teller for whom you will be queuing. Find the probability p that you are the last of the three customers to leave the bank.
(b) Suppose you coose to queue for the quicker teller, find p.
(c) Suppose you go to the teller who is free first. Find p.

Problem 3

Consider an $M/M(k,k)/1$ system. For k = 1,2,3,4,5, compare the mean queue lengths when $\rho = 3/2, 2, 5/2, 3$. (Choose appropriate ρ's in each case.)

Problem 4

Consider a random walk on the non-negative integers with a reflecting barrier at 0 which moves right or left with respective probabilities $a/(1+a)$ and $1/(1+a)$. When at 0, the particle moves to 1 in the next step. Show that the walk has a stationary distribution if and only if $a < 1$ and find these probabilities.

Due date Friday, January 11 2008, 14:00 o'clock. Sheets can be turned in right before class. Please put your **name** and **student id number** on each sheet you turn in and staple the sheets.

Merry Christmas and a happy 2008!