University of Karlsruhe
Institute of Stochastics

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Stochastic Methods in Industry I (WS 07/08)

Problem Set 11

Problem 1

Consider a Jackson network with three nodes. Find the average total time spent in the system by a customer if

$$r_1 = 4, r_2 = 2, r_3 = 1, \mu_1 = 15, \mu_2 = 10, \mu_3 = 6$$

and the transition probability matrix is

$$\begin{pmatrix} 0 & 1/3 & 1/4 \\ 1/2 & 1/4 & 0 \\ 3/8 & 1/4 & 1/8 \end{pmatrix}.$$

Problem 2

Consider a closed queueing network with three servers (nodes) and in total 3 customers in the system (closed in the sense that noone can leave or enter the system). A customer can only move from node i to node $i+1$ $(i = 1, 2)$ and from node 3 to node 1. Assuming the three nodes have service rates μ_1, μ_2, μ_3 respectively, obtain the stationary probabilities for each partition

$$(n_1, n_2, n_3) \in \{0, 1, 2, 3\}^3, \quad \sum n_i = 3$$

of the three customers. Here, n_i is to be interpreted as the number of customers at node i.

Problem 3

Consider a car wash facility where cars arrive according to a Poisson process at the rate of 4 per hour. Assume mean service time is 1/6 hour. Compare the waiting times in the queue when the service times are constant, exponential, Erlangian with 3 phases, uniform between 5 and 15 minutes, discrete with values equal to 2 min and 12 min with respective probabilities 0.2 and 0.8.

Problem 4

In an $M/G/1$ queue, show that the average length of the busy period is $1/(\mu - \lambda)$.

Due date Friday, January 25 2008, 14:00 o'clock. Sheets can be turned in right before class. Please put your **name** and **student id number** on each sheet you turn in and staple the sheets.