University of Karlsruhe
Institute of Stochastics

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Stochastic Methods in Industry I (WS 07/08)

Problem Set 12

Problem 1

In an $M/G/1$ queue let T_n denote the departure time of the n^{th} job and let $X_n := X(T_n+)$ denote the system size right after such times. Show that $X_{n+1} = X_n - H(X_n) + Y_{n+1}$, where Y_n denotes the number of arrivals during the n^{th} customers service time and

$$H(X) = \begin{cases} 1 & \text{if} \quad X > 0 \\ 0 & \text{if} \quad X = 0 \end{cases}.$$

Obtain the Pollaczek-Khinchin formula

$$L_q = \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)} \quad \text{(standard notations from class)}$$

by squaring both sides, taking expectations and investigating the limit $n \to \infty$.
(You may use without proof that $\lim_{n\to\infty} \mathbb{E}X_n = \mathbb{E}X < \infty$ and $\lim_{n\to\infty} \mathbb{E}Y_n = \mathbb{E}Y < \infty$, where X and Y are the random system size and the random number of arrivals during one service in steady state. Also, observe that $H^2(X) = H(X)$ and $XH(X) = X$.)

Problem 2

A cellular radio telephone system serves a given geographical area with $2m$ telephone channels connected to a single switching centre. There are two types of calls: radio-to-radio calls, which occur with a Poisson rate λ_1 and require two radio channels per call, and radio-to-non-radio calls, which occur at a Poisson rate λ_2 and require one radio channel per call. The duration of all calls is exponentially distributed with mean $1/\mu$. Calls that cannot be accommodated by the system are blocked. Give formulas for the blocking probability of the two types of calls.

Problem 3

Persons arrive at a Xerox machine according to a Poisson process with rate 1 per minute. The number of copies to be made by each person is uniformly distributed between 1 and 10. Each copy requires 3 seconds. Find the average waiting time in the queue when
(a) each person uses the machine on a FCFS basis
(b) Persons with no more than 2 copies to make are given non-preemptive priority over the other persons.

Problem 4

For a $D/M/1$ queue, compute L_s for $\rho = \frac{1}{10}, \frac{2}{10}, \ldots, \frac{8}{10}, \frac{9}{10}$.

Due date Friday, February 1st 2008, 14:00 o'clock. Sheets can be turned in right before class. Please put your **name** and **student id number** on each sheet you turn in and staple the sheets.