Karlsruhe Institute of Technology

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Stochastic Processes
Problem sheet 1

Problem 1
Let P be a stochastic matrix and (Y_n) a sequence of independently $\mathcal{U}(0,1)$-distributed random variables. Show that there is a function g s.t. (X_n) defined by $X_n = g(X_{n-1}, Y_n)$ is a Markov chain with transition matrix P.

Problem 2
Let the generation 0 of a population consist of one individual, which splits itself up to k descendants with probability p_k ($k \in \mathbb{N}$). Every one of those descendants (generation 1) splits up again to a random number of descendants according to the distribution $(p_k)_{k\in\mathbb{N}}$. Let the random variable X_n describe the number of individuals in the n-th generation ($n \in \mathbb{N}$).
Show that (X_n) is a Markov chain and determine the transition matrix.

Problem 3
Prove or disprove the following assertion:

If (X_n) is a Markov chain with values in S, we have

$$P(X_{n+1} = j \,|\, X_0 \in A_0, \ldots, X_n \in A_n) = P(X_{n+1} = j \,|\, X_n \in A_n)$$

for all $j \in S$ and $A_1, \ldots, A_n \subseteq S$ with $P(X_0 \in A_0, \ldots, X_n \in A_n) > 0$.

Problem 4
A company offers their field staff various career opportunities, given adequate commitment. There is a total of 7 salary levels, where level 7 assures a lifetime employment. A field worker from level i ($i \leq 6$) can be promoted to level $i+1$ or leaves the company due to a better offer by a competing company ("level 8"). The typical field worker's career proceeds according to a Markov chain with transition matrix P given by:

$$P = \begin{pmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0.6 & 0 & 0 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.8 & 0 & 0.2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.01 & 0.99 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Determine the communicating classes and calculate f^*_{17}.

This sheet will be discussed in the problem sessions on the 21st of April.