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Stochastic Processes
Problem sheet 2

Problem 1
Let (X_n) be a homogenous Markov chain with state space $S = \{0, 1, 2\}$ and transition matrix

$$A := \begin{pmatrix} 0 & 1 & 0 \\ 1-p & 0 & p \\ 0 & 1 & 0 \end{pmatrix}$$

with $0 \leq p \leq 1$.

a) Calculate $p_{ij}^{(n)}$ for $i, j \in S$ and $n \in \mathbb{N}$.

b) Specify the recurrent states dependent on p.

Problem 2
Let $(X_n)_{n\in\mathbb{N}}$ be the branching process from sheet 1 problem 2. Thus $X_n = X_{n,1} + \ldots + X_{n,X_{n-1}}$, where the random variables $X_{n,j}$ $(n, j \in \mathbb{N})$ are i.i.d with $P(X_{n,j} = k) = p_k$ for $k \in \mathbb{N}$. Specify the recurrent and transient states.

Problem 3
Let $\mathcal{P}(m)$ denote the Poisson distribution with parameter m. We draw random numbers from $\mathbb{N}$ by the following procedure:
First we draw a number N_1 according to the distribution $\mathcal{P}(1)$. Then we draw a number N_2 according to the distribution $\mathcal{P}(N_1)$. The k-th number N_k is drawn according to the distribution $\mathcal{P}(N_{k-1})$.
Show that with probability 1 there is a 0 in the sequence $(N_k)_{k\in\mathbb{N}}$.

Problem 4
Prove Lemma 2.15:

If $i \in S$ and $j \in S$ are in the same recurrent class, we have $f_{ij}^* = f_{ji}^* = 1$.

This sheet will be discussed in the problem sessions on the 28th of April.