Karlsruhe Institute of Technology

Institute for Stochastics
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Stochastic Processes
Problem sheet 6

Problem 1
Let N be a Poisson process with parameter λ. Show that the Flip-Flop process $X_t := (-1)^{N_t}$ is a continuous-time Markov chain (CTMC) and determine the transition probabilities.

Problem 2
Let $\{p(t) : t \geq 0\}$ be a standard transition semigroup (STS) on S. Show that for all $i, j \in S$ the function $t \mapsto p_{ij}(t)$ is uniformly continuous.

Problem 3
Let $\{p(t) : t \geq 0\}$ be an irreducible STS on S (i.e. $\{p(t) : t \geq 0\}$ is an STS on S and for all $i, j \in S$ there is some $t > 0$ with $p_{ij}(t) > 0$).

Show that for any $h > 0$ the matrix $P_h := p(h)$ is the irreducible and aperiodic transition matrix of a discrete-time Markov chain.

Problem 4
Let $\{p(t) : t \geq 0\}$ be an irreducible STS.
Let the probability vector $\pi = (\pi_i)_{i \in S}$ be stationary for every $P \in \{p(t) : t \geq 0\}$. Show, that in this case

$$\lim_{t \to \infty} p_{ij}(t) = \pi_j \quad \text{for all } i, j \in S.$$

This sheet will be discussed in the problem sessions on the 26th of May.

Please note that the English tutorial has been moved to Wednesday 11:30-13:00, Plank-Hörsaal (40.32)