Karlsruhe Institute of Technology

SPP2265 Random Geometric Systems

New Trends in Point Process Theory

International Workshop

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SPP2265: Random Geometric Systems

28 Feb - 2 Mar 2022
Karlsruhe, Germany

Organisation
Steffen Betsch & Günter Last

https://www.math.kit.edu/stoch/event/ppt2022/

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About

New trends in point process theory

The first topic of the workshop are Gibbs point processes. Recent years have seen some progress on phase transitions of such processes, and even huge progress in their distributional analysis (decorrelation, central limit theorems, extreme value theory).

The second topic of the workshop is hyperuniformity, an amazing property of lattice-like but yet random systems. The workshop will deal in particular with characterizations of hyperuniformity, rigidity, and examples.

The workshop is supported by the DFG Priority Programme SPP 2265: Random Geometric Systems.

Organization

Steffen Betsch and Günter Last

Technical support

Tatjana Dominic

The website for this workshop can be found at https://www.math.kit.edu/stoch/seite/ppt2022/.

Schedule

Monday, February 28

8:40–9:00	**Welcome remarks**	
9:00–9:55	**Steffen Betsch** Karlsruhe Institute of Technology	On the uniqueness of Gibbs distributions with a non-negative and subcritical pair potential
9:55–10:30	**Coffee break**	
10:30–11:25	**Sabine Jansen** LMU Munich	Cluster expansions - Part I
11:30-12:25	**Leonid Kolesnikov** LMU Munich	Cluster expansions - Part II
12:25–14:30	**Lunch break**	
14:30–15:25	**Alexander Zass** WIAS Berlin	Marked Gibbs point processes: a path space example
15:30–16:25	**Moritz Otto** University of Magdeburg	Couplings and Poisson approximation of functionals of Gibbs processes
16:25–17:00	**Coffee break**	
17:00–17:55	**Christian Hirsch** Aarhus University	CLTs for the persistence diagram on Gibbsian tessellations

Tuesday, March 1

9:00–9:55	**Hartmut Löwen** University of Düsseldorf	Nonreciprocal interactions
9:55–10:30	**Coffee break**	
10:30–11:25	**Alexey Bufetov** University of Leipzig	Asymmetric exclusion process via Mallows coloring
11:30–12:25	**Raphaël Lachieze-Rey** University Paris Descartes	Variance linearity for Gaussian nodal domains
12:25–14:00	**Lunch break**	
14:00–15:30	**Excursion**	
16:00–16:55	**René Wittmann** University of Düsseldorf	Uniqueness and hyperuniformity of positional density profiles for active particles
16:55–17:30	**Coffee break**	
17:30–18:25	**Diala Hawat** Université de Lille	Exploring the hyperuniformity of a point process using its structure factor with Python
19:00	**Conference Dinner**	

Wednesday, March 2

9:00–9:55	**Matthias Schulte** Hamburg University of Technology	Large degrees in scale-free inhomogeneous random graphs
9:55–10:30	**Coffee break**	
10:30–11:25	**Martin Huesmann** University of Münster	Bipartite matching, invariance, and regularity of optimal transport
11:30–12:25	**Tobias Kuna** University of L'Aquila	Realizability problem for point processes
12:25	**Time to say goodbye**	

List of Abstracts – Talks

Betsch – Mon. 28th (9:00 - 9:55)

On the uniqueness of Gibbs distributions with a non-negative and subcritical pair potential

Steffen Betsch

Karlsruhe Institute of Technology, Germany

We prove that the distribution of a Gibbs process with non-negative pair potential is uniquely determined as soon as an associated Poisson-driven random connection model (RCM) does not percolate. Our proof combines disagreement coupling in continuum with a coupling of a Gibbs process and a RCM. The improvement over previous uniqueness results is illustrated both in theory and simulations.

The talk is based on joint work with Günter Last: arXiv:2108.06303

Jansen and Kolesnikov – Mon. 28th (10:30 - 12:25)

Cluster expansions (Part I and II)

Sabine Jansen and Leonid Kolesnikov

Ludwig Maximilian University of Munich, Germany

Cluster expansions are a commonly used tool in perturbative approaches to Gibbs point processes with pairwise interactions. We prove a new convergence criterion for the activity expansion of Gibbs correlation functions. For non-negative pair potentials, the criterion is an if and only if condition. It can be formulated via a sign-flipped Kirkwood-Salsburg operator and known conditions such as Kotecký-Preiss and Fernández-Procacci are easily recovered. Moreover, the criterion provides an approach to find new convergence conditions in both discrete and continuous setups — we discuss examples for abstract polymer systems and for multi-type hard spheres in $\mathbb{R}^d$.

The talk is based on the following preprint: arXiv:2112.13134v2.

Zass – Mon. 28th (14:30 - 15:25)

Marked Gibbs point processes: a path space example

Alexander Zass

Weierstrass Institute for Applied Analysis and Stochastics, Germany

The motivation for this work comes from seeing a class of infinite-dimensional diffusions under Gibbsian interactions as marked point configurations: the starting points belong to $\mathbb{R}^d$, the marks are the paths of Langevin diffusions, and the interaction between two diffusions is given by the integration of a pair potential along their paths.

In this talk, after presenting this example, we prove the existence of an infinite-volume Gibbs point process, by applying a general entropy method result to this setting of pair potentials, and then provide an explicit activity domain in which uniqueness holds, by using cluster expansion tools.

Otto – Mon. 28th (15:30 - 16:25)

Couplings and Poisson approximation of functionals of Gibbs processes

Moritz Otto

University of Magdeburg, Germany

I will present a bound on the Kantorovich-Rubinstein distance between an appropriately scaled thinning of a marked Gibbs process with bounded Papangelou intensity and a Poisson process. The bound is based on a coupling technique for Poisson approximation and exploits the disagreement coupling for a Gibbs process and its Palm version. It is assumed that a random graph defined on the points of a dominating Poisson process does not percolate. As an application of our general findings, I will discuss Poisson approximation for Matérn type I thinnings of Gibbs processes.

The talk is based on joint work with Günter Last: arXiv:2104.00737

Hirsch – Mon. 28th (17:00 - 17:55)

CLTs for the persistence diagram on Gibbsian tessellations

Christian Hirsch

Aarhus University, Denmark

In applications in materials science, it is common to work with tessellation data where the cell centers are not scattered entirely at random but are subject to repulsive interactions. Therefore, Gibbs-Voronoi and Gibbs-Laguerre tessellations are important building blocks when constructing stochastic geometry models. Moreover, recently the persistence diagram has become a popular tool to detect subtle topological features in data. Based on the framework in (Schreiber & Yukich, 2013), I will present a functional CLT for the persistence diagram on Gibbsian tessellations, which can form the basis for goodness-of-fit tests. I will also elaborate on how persistence vineyards can be used to design rigorous statistical hypothesis tests for 3D microstructure models based on data from 2D slices.

This talk is based on joint work with A. Cipriani, J. Krebs, C. Redenbach, and M. Vittorietti.

Löwen – Tue. 1st (9:00 - 9:55)

Nonreciprocal interactions

Hartmut Löwen

University of Düsseldorf, Germany

Bufetov – Tue. 1st (10:30 - 11:25)

Asymmetric exclusion process via Mallows coloring

Alexey Bufetov

University of Leipzig, Germany

Asymmetric simple exclusion process (ASEP) is one of the most studied models of interacting particle systems. It is known to belong to the Kardar-Parisi-Zhang universality class in particular due to pioneering work of Johansson and Tracy-Widom. In this talk I will review these classical results and also discuss new asymptotic results obtained by considering the multi-species (or multi-color) extension of the process. The key role will be played by the so-called Mallows measure on infinite permutations, which is due to Gnedin-Olshanski.

Lachieze-Rey – Tue. 1st (11:30 - 12:25)

Variance linearity for Gaussian nodal domains

Raphaël Lachieze-Rey

University Paris Descartes, France

Given a real Euclidean stationary Gaussian process observed in a growing window, we investigate whether the variance of the Gaussian excursion or level set measure is proportional or negligible to the window volume. In dimension 1, it turns out that the variance of the number of zeros on a large interval is always at least proportional to the interval length, meaning the zero process is not hyperuniform; we further investigate conditions for variance linearity, and exhibit examples experiencing rigidity. In higher dimensions, we investigate the excursion volume (or defect volume), and prove that under isotropy conditions it is not hyperuniform either, reproving a variance cancellation phenomenon already observed on the Sphere. If we remove the isotropy constraint, we study diophantine spectral measures and exhibit excursion sets with arbitrary hyperuniformity index; this goes through studying random walks with irrational increments on the torus.

Wittmann – Tue. 1st (16:00 - 16:55)

Uniqueness and hyperuniformity of positional density profiles for active particles

René Wittmann

University of Düsseldorf, Germany

We want to investigate the uniqueness of the mapping between external physical fields and the corresponding density profiles for the particle positions in colloidal fluids. In such a system, the motion of the particles is described by stochastic processes. In particular, we consider "passive" Brownian particles, driven by a Gaussian white noise, and "active" particles, driven by more general noises which induces non-Markovian dynamics. In this talk, we give a short overview of the physics of active particles and establish desired uniqueness relations by calculating the exact stationary probability density. After recalling the equilibrium result for noninteracting passive particles in an external potential (inhomogeneous Poisson process), we develop and solve a suitable model for noninteracting active particles in an external physical field inducing a position-dependent "activity". Given these rigorous results, we further discuss the possibility to transfer hyperuniformity to the density profile by imposing an appropriate (hyperuniform) external physical field.

Hawat – Tue. 1st (17:30 - 18:25)

Exploring the hyperuniformity of a point process using its structure factor with Python

Diala Hawat

Université de Lille, France

Condensed matter physicists have observed that, for some random particle systems, the variance of the number of points in a large window scales slower than the volume of that window, a phenomenon called hyperuniformity. Hyperuniform point processes (HU) have generated broad interest in physics, statistics, machine learning, and probability. By definition, a hyperuniform point process leads to a fast-decaying Monte Carlo error when estimating volumes. There are many candidate HU processes in the physics literature, but rigorously proving that a point process is HU is usually difficult. It is thus desirable to have standardized numerical tests of hyperuniformity. The Common practice in statistical physics is to estimate a spectral measure called the structure factor, the behavior of which around zero is a sign of hyperuniformity. We survey spectral estimators of the structure factor using tapering and multitapering methods with other isotropic estimators used in the literature for different study purposes, we study their efficiency in the context of hyperuniformity and gather them all in a Python toolbox, along with a numerical diagnosis of hyperuniformity.

Github link: https://github.com/For-a-few-DPPs-more/structure-factor
PyPi link: https://pypi.org/project/structure-factor/
Documentation link: https://for-a-few-dpps-more.github.io/structure-factor/

Schulte – Wed. 2nd (9:00 - 9:55)

Large degrees in scale-free inhomogeneous random graphs

Matthias Schulte

Hamburg University of Technology, Germany

Large graphs with a highly non-trivial structure, so-called complex networks, arise in many different fields, ranging from natural to social sciences. Prominent examples are the internet, the World Wide Web or social networks. Such complex networks can be modelled by random graphs. We consider a class of random graphs whose construction involves weights and whose degree distributions follow power-laws as observed for many real-world complex networks. Examples are some long-range percolation models, the random connection model with weights, the Norros-Reittu model and the Chung-Lu model. For such random graphs we study the maximum degree in a growing observation window and show that its limiting distribution is Frechet. More generally, we establish that the point process of large degrees converges in distribution to an inhomogeneous Poisson process on the positive half-line. An important statistical question is to estimate the tail exponent of the degree distribution. Here we prove consistency of the Hill estimator.

This talk is based on joint work with Chinmoy Bhattacharjee (University of Luxembourg).

Huesmann – Wed. 2nd (10:30 - 11:25)

Bipartite matching, invariance, and regularity of optimal transport

Martin Huesmann

University of Münster, Germany

The bipartite matching problem is one of the classical random optimization problems. The macroscopic behaviour is well understood since the work of Ajtai, Komlos, Tusnady, Talagrand and others in the 80s and early 90s. A few years ago, Caracciolo et al. proposed a new ansatz, based on a linearisation of the Monge-Ampère equation to the Poisson equation, to get refined estimates for this problem. I will show how one can use their ansatz to prove that in dimension two certain thermodynamic limits of the optimal bipartite matchings do not exist. A key tool is a harmonic approximation result for optimal couplings between arbitrary measures.

This is based on joint work with Michael Goldman, Francesco Mattesini, and Felix Otto.

Kuna – Wed. 2nd (11:30 - 12:25)

Realizability problem for point processes

Tobias Kuna

University of L'Aquila, Italy

In the talk an overview is given about the so-called realizability problem. The latter considers the question whether a given set of functions can be realised as the factorial moments (correlation functions) of a point process. In particular, we review general existence result and its relation to a particular notion of positivity. Furthermore, we present explicit constructions for moderate low density. In passing, we describe motivations from applications for this problem.

The work presented is joint with Maria Infusion (Calgiari), Joel Lebowitz (Rutgers), Eugene Speer (Rutgers).

List of Participants

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