Preface

The Poisson process generates point patterns in a purely random manner. It plays a fundamental role in probability theory and its applications, and enjoys a rich and beautiful theory. While many of the applications involve point processes on the line, or more generally in Euclidean space, many others do not. Fortunately, one can develop much of the theory in the abstract setting of a general measurable space.

We have prepared the present volume so as to provide a modern textbook on the general Poisson process. Despite its importance, there are not many monographs or graduate texts with the Poisson process as their main point of focus, for example by comparison with the topic of Brownian motion. This is probably due to a viewpoint that the theory of Poisson processes on its own is too insubstantial to merit such a treatment. Such a viewpoint now seems out of date, especially in view of recent developments in the stochastic analysis of the Poisson process. We also extend our remit to topics in stochastic geometry, which is concerned with mathematical models for random geometric structures [4, 5, 23, 45, 123, 126, 147]. The Poisson process is fundamental to stochastic geometry, and the applications areas discussed in this book lie largely in this direction, reflecting the taste and expertise of the authors. In particular, we discuss Voronoi tessellations, stable allocations, hyperplane processes, the Boolean model and the Gilbert graph.

Besides stochastic geometry, there are many other fields of application of the Poisson process. These include Lévy processes [10, 83], Brownian excursion theory [140], queueing networks [6, 149], and Poisson limits in extreme value theory [139]. Although we do not cover these topics here, we hope nevertheless that this book will be a useful resource for people working in these and related areas.

This book is intended to be a basis for graduate courses or seminars on the Poisson process. It might also serve as an introduction to point process theory. Each chapter is supposed to cover material that can be presented (at least in principle) in a single lecture. In practice, it may not always be possible to get through an entire chapter in one lecture; however, in most chapters the most essential material is presented in the early part of the chapter, and the later part could feasibly be left as background reading if necessary. While it is recommended to read the earlier chapters in a linear order at least up to Chapter 5, there is some scope for the reader to pick and choose from the later chapters. For example, a reader more interested in stochastic geometry could look at Chapters 8–11 and 16–17. A reader wishing to focus on the general abstract theory of Poisson processes could look at Chapters 6, 7, 12, 13 and 18–21. A reader wishing initially to take on slightly easier material could look at Chapters 7–9, 13 and 15–17.

The book divides loosely into three parts. In the first part we develop basic results on the Poisson process in the general setting. In the second part we introduce models and results of stochastic geometry, most but not all of which are based on the Poisson process, and which are most naturally developed in the Euclidean setting. Chapters 8, 9, 10, 16, 17 and 22 are devoted exclusively to stochastic geometry while other chapters use stochastic geometry models for illustrating the theory. In the third part we return to the general setting and describe more advanced results on the stochastic analysis of the Poisson process.

Our treatment requires a sound knowledge of measure-theoretic probability theory. However, specific knowledge of stochastic processes is not assumed. Since the focus is always on the probabilistic structure, technical issues of measure theory are kept in the background, whenever possible. Some basic facts from measure and probability theory are collected in the appendices.

When treating a classical and central subject of probability theory, a certain overlap with other books is inevitable. Much of the material of the earlier chapters, for instance, can also be found (in a slightly more restricted form) in the highly recommended book [75] by J.F.C. Kingman. Further results on Poisson processes, as well as on general random measures and point processes, are presented in the monographs [6, 23, 27, 53, 62, 63, 69, 88, 107, 134, 139]. The recent monograph Kallenberg [65] provides an excellent systematic account of the modern theory of random measures. Comments on the early history of the Poisson process, on the history of the main results presented in this book and on the literature are given in Appendix C.

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