Program and Abstracts of the International Conference

Geometry and Physics
of Spatial Random Systems

Bad Herrenalb, 11-15 September 2017

An interdisciplinary exploration between mathematics and physics of systems with disordered spatial structure
Dear Colleagues,

Welcome to Bad Herrenalb and thank you for joining “Geometry and Physics of Spatial Random Systems”!

The conference borrows its name from a research unit composed of university departments from Karlsruhe, Erlangen and Aarhus. The activities of this research unit, founded in 2011 and funded by the German Research Foundation, revolve around spatial systems with disorder, where advanced concepts from stochastic and integral geometry are essential for an in-depth understanding of the physical properties. This question of spatial disorder is also the unifying theme of most contributions detailed in this booklet, covering diverse topics such as cellular matter, random packings, random fields, percolation, tensor valuations, Boolean models and random tessellations, stereology, etc.

As is the case for the research unit, our vision for the conference is to be an interdisciplinary exploration of disordered spatial structure, with lively and engaging discussion between mathematics and physics, and related disciplines. We hope that you will in particular benefit from the presentations that are not directly in your area of expertise. Please bear this hope in mind also during your own presentation, and please do not shy away from asking questions when out of your depth.

Enjoy your week in the Black Forest!

Daniel Hug, Eva Vedel Jensen, Markus Kiderlen, Günter Last, Klaus Mecke, Gerd Schröder-Turk, Wolfgang Weil and Steffen Winter
GENERAL INFORMATION

Conference Venue

Our conference venue Haus der Kirche is a conference center of the Protestant Church of Germany. It is situated at Bad Herrenalb, a health resort in the northern part of the Black Forest. The town centre with its spa gardens, mineral springs and some remains of a medieval monastery as well as the train station are all within walking distance from the conference venue. The street address is Evangelische Akademie Baden, Dobler Str. 51, 76332 Bad Herrenalb, Germany.

For a street map of Bad Herrenalb see the inside of the front cover.

Meals, Drinks and Conference Dinner

Breakfast is served from 7.30 to 9.00am.

Lunch is served at 12:30 und dinner at 6.30pm.

During coffee breaks water, coffee and tea will be available. Water will also be available during all meals. Other drinks are available at any time but are not covered. There are several fridges and shelves with drinks throughout the house with an honesty box beside them. It is also possible to get a sheet of paper from the reception desk on which you can mark your consumed drinks and pay in the end.

If you have any special dietary requirements or food allergies, please discuss these directly with the hotel staff or come and talk to us.

On Thursday evening you are cordially invited to a conference dinner barbecue starting at 7:30pm to celebrate a hopefully useful and enjoyable conference.

Wireless Internet Access

Wireless internet access is provided free of charge for all participants within the hotel. The name of the network is HdK-Hotspot and the network key is HdK47110.
General Information

Session Format and Talk Style

For the sake of lively discussion, please respect the following maximal talk times allocated to your presentations:

Keynote lectures: 40 minutes talk time + discussion
Invited talks: 30 minutes talk time + discussion
Contributed talks: 20 minutes talk time + discussion

Times allocated to the sessions are generous to allow for in-depth questions and extended discussion.

When preparing and presenting your talk, please bear in mind that the audience will be composed of mathematicians and physicists, in roughly equal proportion. We hope for presentations by the mathematicians that in particular also engage the physicists, and vice versa, to encourage lively discussion across the discipline boundaries. Where possible, please make your presentation accessible for both communities.

Poster Session

Posters are displayed for the duration of the conference. There is a dedicated Poster Session on Tuesday from 8pm to 10pm. Abstracts for posters can be found in this booklet starting from page 93.

Please take note of the Best Poster Award. Your conference booklet contains a ballot slip for your choice of the three best posters. Please hand in your completed ballot slip by the coffee break on Wednesday morning.
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Fast approximation of the intensity of a spatial point process

Adrian Baddeley$^{1,2,*}$ and Gopalan Nair$^{3,2}$

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A classical problem in statistical physics is to determine macroscopic properties of a gas, such as its average density, from a Gibbs specification of the stochastic interactions between individual molecules. Exact analytic solutions are rare, and the best available answer is usually an approximation. Common techniques include the mean-field and Poisson-Boltzmann-Emden approximations [1, 2].

This problem has implications for the field of spatial statistics, where Gibbs models have been enlisted as statistical models to be fitted to spatial data [3, Chap. 13]. For example, the observed spatial locations of plants in a meadow could be modelled as a Gibbs spatial point process, in which the stochastic interactions describe the effect of competition between individual plants, and an external field would describe spatial variation in soil fertility. After a Gibbs model is fitted to data, it must be validated by statistical procedures such as goodness-of-fit tests; but these often require calculation of the intensity (average density) $\lambda$ of the fitted model as a function of the model parameters.

In [4–6] we developed a simple technique for approximating $\lambda$, by interpreting a well-known identity for Gibbs processes as a kind of self-consistency equation for $\lambda$. Replacing one side of this equation by a simpler expression, we solve the modified equation to obtain an approximation to $\lambda$ which we call the Poisson-saddlepoint approximation.

For pairwise interaction Gibbs point processes, the Poisson-saddlepoint approximation is equivalent to the Poisson-Boltzmann-Emden approximation of statistical physics [1, 2]. In this case the self-consistency equation can be stated explicitly, and the approximation can be calculated analytically if the model is stationary.

For higher-order interactions, the Poisson-saddlepoint approximation does not seem to be equivalent to existing approximations. Examples of such processes include the Widom-Rowlinson penetrable sphere model [7, 8] and the Geyer [9] saturation model, both of which have infinite order of interaction. In principle, the Poisson-saddlepoint approximation requires knowledge of the moment ge-

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generating function $M_\lambda(t)$ of the canonical sufficient statistic of the model with respect to a Poisson point process of intensity $\lambda$, for all $\lambda$. Typically this is an unsolved problem in stochastic geometry.

In [10] we developed a practical approach that uses scaling properties, limit theorems from stochastic geometry, and small-sample probabilities estimated once-and-for-all by an extensive simulation study, to form an approximation to the moment generating function, and thereby to approximate the intensity.

A brief sketch of the technique follows. We start with the identity

$$\lambda = \mathbb{E}[\Lambda_X(0, X)], \quad (1)$$

where $X$ is a Gibbs point process (assumed to be stationary for simplicity in this abstract), $\lambda$ is its intensity, and $\Lambda_X(u, X)$ is the Papangelou conditional intensity of $X$ at location $u$. Eq. (1) is a special case of the Georgii-Nguyen-Zessin formula [11–14].

On the right hand side of Eq. (1), typically the conditional intensity $\Lambda_X(u, X)$ has a simple form determined by the Gibbs model, but the expectation is with respect to the unknown probability distribution of the Gibbs process $X$, so the right hand side of Eq. (1) is intractable.

The *Poisson-saddlepoint approximation* of $\lambda$ is the solution $\lambda^{PS}$, if it exists uniquely, of

$$\lambda = \mathbb{E}[\Lambda_X(0, \Pi_\lambda)], \quad (2)$$

where $\Pi_\lambda$ is the homogeneous Poisson point process in $\mathbb{R}^d$ with intensity $\lambda$. The expectation on the right-hand side of Eq. (1) has been replaced in Eq. (2) by the expectation with respect to a Poisson process with the same unknown intensity.

For a pairwise-interaction Gibbs process with (exponentiated) pair potential $g(u, v) = g(||u - v||)$, the Poisson-saddlepoint approximation of the intensity of $X$ is [4]

$$\lambda^{PS} = \frac{W(\beta G)}{G}, \quad (3)$$

where $W$ is Lambert’s $W$-function [15], the inverse function of $z \mapsto ze^z$, and

$$G = \int_{\mathbb{R}^d} [1 - g(v)] \, dv. \quad (4)$$

The approximation Eq. (3) is identical to the Poisson-Boltzmann-Emden approximation [1, 2]. The approximation is surprisingly good in many examples, though not in all [16, 17]. Stucki and Schuhmacher [17] obtained a bound on the absolute error.

Now consider a Gibbs model with interpoint interactions of any order, such as the Widom-Rowlinson penetrable spheres model. Evaluation of the right-hand side of Eq. (2) is now a problem in stochastic geometry.
For a Gibbs model with finite range of interaction $R$, we have $\Lambda_X(0; x) = \Lambda_X(0; x \cap b(0, R))$ where $b(0, R)$ is the ball of radius $R$ centred at the origin.

The number of points in $X \cap b(0, R)$ is almost surely finite; conditioning on the number of points gives

$$\mathbb{E}[\Lambda_X(0, \Pi_\lambda)] = \sum_{n=0}^{\infty} p_\mu(n) M_{n,R},$$

where $p_\mu(n) = e^{-\mu} \mu^n / n!$ is the probability of $n$ points for a Poisson distribution with mean $\mu = \lambda R^d \omega_d$, and

$$M_{n,R} = \mathbb{E}[\lambda X(0, Z_{n,R})],$$

where $Z_{n,R}$ is the binomial point process consisting of exactly $n$ independent random points uniformly distributed in $b(0, R)$. If the model has suitable scaling properties, it suffices to assume $R = 1$. In [10] we propose the following strategy: for small $n$, we evaluate $M_{n,1}$ numerically, once-and-for-all, using a large simulation experiment. For large $n$, we use limit theorems from stochastic geometry to approximate $M_{n,1}$ analytically. These approximations are collected via Eq. (5) into a numerical approximation to $M(\lambda) = \mathbb{E}[\Lambda_X(0, \Pi_\lambda)]$, and we solve the equation $\lambda = M(\lambda)$ numerically to obtain the Poisson-saddlepoint approximation $\lambda^{PS}$.

This strategy requires the existence of limit results for the canonical sufficient statistic of the model. For the Widom-Rowlinson model, this is a problem about random coverage by unions of spheres; several asymptotic results are available [18–21] and can be pressed into service.

Approximations have been implemented in software [3] for the Widom-Rowlinson and Geyer models, and perform quite well [10]. It would be of great interest to extend the results of Schuhmacher and Stucki [16, 17] to higher-order interactions, to give bounds on the accuracy of the approximation.

**ACKNOWLEDGMENTS**

Adrian Baddeley was supported by the Australian Research Council (Discovery Grant DP 130104470, ARC Discovery Outstanding Researcher Award).

Talks (Monday 11 September 2017)

Analysis of the Dynamics of Complex Spatio-Temporal Systems from Time Series

Miroslav Kramar

Tohoku University, Japan

In this talk we will demonstrate applications of topological data analysis to granular systems. The force bearing structures (force chains) present in dense granular systems form complicated patterns that often evolve in time. The dynamics of the pattern formation present in dense granular media is extremely complicated. In order to study the dynamic of these systems we use persistence diagrams to describe the state of the system. This procedure allows us to transform experimental or numerical data, from experiment or simulation, into a point cloud in the space of persistence diagrams. There are a variety of metrics that can be imposed on the space of persistence diagrams to obtain a deeper insight into the dynamics of the process of pattern formation. We will use these metrics to identify the important time scales at which the behaviour of the system changes. We will also discuss a physical interpretation of these time scales.

In the second part of the talk we will discuss the deformation patterns of silo walls during gravity-driven granular discharges. Depending on the initial filing height and/or protocol the silos might undergo serious deformations leading to a potential collapse. We will show that our methods can clearly distinguish the collapsing silos during the early stages of the discharge.
Universal hidden order in amorphous cellular geometries

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Partitioning space into cells [1] is central to many fields of science and technology, as well as to resource distribution problems in economics and telecommunication. The nature of such cellular partitions is often defined by optimization with respect to certain properties, such as interface area in the Kelvin problem [2], packing density in the Kepler problem [3], or cell centrality as in the Quantizer problem [4]. In all known cases, the optimal solutions are crystalline configurations with long range order. Amorphous disordered structures are generally considered to be intermittent meta-stable states that prevent the system from attaining the optimal ordered structures. To date, no optimization problem has

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been identified where the optimal solution is a disordered configuration. In this talk, we will use Lloyd’s algorithm [5], see Fig. 1, to show that a stable disordered state exists in the three-dimensional Quantizer problem, despite the existence of lower-energy crystalline configurations. Akin to a thermodynamic phase, this state is universal; i.e., irrespective of the level and type of disorder in the initial configurations, we find a convergence to the same amorphous state, representing configurations characterized by the same structure factor and energy distributions. This highly degenerate state is characterised by an anomalous suppression of long-wavelength density fluctuations, known as hyperuniformity [6]. A clear signature of the dimension of the ambient space is revealed: In 3D, the universal hyperuniform state is fully disordered across length scales without any trace of local crystalline structure, see Fig. 2. By contrast, while the 2D system has a greater predisposition to form locally ordered domains, as expected, it remarkably remains globally amorphous and is effectively hyperuniform. For systems related to the Quantizer problem, such as self-assembled copolymeric phases, our findings suggest the possibility of stable disordered hyperuniform phases.

ACKNOWLEDGMENTS

We acknowledge the support of the German Science Foundation (DFG) through the research group “Geometry and Physics of Spatial Random Systems” (GPSRS) under grants number SCHR-1148/3-2, HU1874/3-2, and LA965/6-2 and of the European Research Council (ERC) under grant ERC StG MembranesAct 2013-
Fig. 2: Convergence of Lloyd’s iteration in 3D: Shown is a subset of a highly disordered and clustering 3D system after $N = 0$, $N = 5$ and $N = 12000$ iterations of Lloyd’s iteration. The system develops a global uniformity but remains amorphous. The energy of the converged tessellation $\langle e \rangle \approx 1.008 \times e_{BCC}$ is slightly above the value $e_{BCC} = 0.07854$ of the optimal crystalline BCC lattice.

33728. We thank the German Academic Exchange Service and Universities Australia for travel funding through a collaborative grant scheme. SK, MK, and GST are grateful to Klaus Mecke (Erlangen) for years of support, moral and financial, and for scientific guidance and inspiration without which this article would have never materialized.

Variation of the Nazarov-Sodin constant for random plane waves and arithmetic random waves.

Igor Wigman

King’s College London*

This talk is based on a joint work with Pär Kurlberg. Let $\mathcal{M}$ be a smooth compact surface (with or without a boundary). For a smooth function $g : \mathcal{M} \to \mathcal{R}$ the nodal components are the connected components of the zero set $g^{-1}(0)$, and the nodal domains are the connected components of the complement $\mathcal{M}\setminus g^{-1}(0)$. We are interested in the number of nodal domains (equivalently, components) of Laplace eigenfunctions on $\mathcal{M}$ in the high energy limit (“nodal count”), i.e. functions $\varphi_j$ satisfying

$$\Delta \varphi_j + \lambda_j^2 \varphi_j = 0,$$

$\lambda_j \to \infty$ (Dirichlet or Neumann boundary conditions), where $\{\varphi_j\}_{j \geq 1}$ is an orthonormal basis of $L^2(\mathcal{M})$.

It was conjectured that the nodal count is related to the analogous problem for Gaussian random fields on $\mathcal{R}^2$. Let $F : \mathcal{R}^2 \to \mathcal{R}$ be a Gaussian random field. Motivated by study of the nodal structures of Laplace eigenfunctions on arbitrary

\[ Left: \text{Nodal picture for a (randomly drawn) spherical harmonic of degree 80, by A. Barnett.} \]
\[ Right: \text{Nodal structures for a particularly chosen toral eigenfunction.} \]

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surfaces we are interested in the random variable $N(F; R)$ of the number of nodal domains (or components) lying entirely inside a big ball $B(R)$, $R \to \infty$. For a wide variety of random fields $F$ it was proven by Nazarov-Sodin that there exists a constant $c = c_{NS}(F) \geq 0$ ("Nazarov-Sodin constant") such that $N(F; R) = c \cdot R^2 + o(R^2)$ (with stronger convergence in mean), and under a slightly more restrictive condition $c > 0$ is strictly positive.

We consider $c_{NS}(F) = c_{NS}(\rho)$ as a functional of the field (or, what is equivalent, the spectral density $\rho$ of $F$). We generalise $c_{NS}(\rho)$ for a somewhat wider class of all fields corresponding to spectral measures supported in the unit ball $B(1)$ (or any other compact domain), and prove that, considered equipped with the weak-$\ast$ topology on these measures $c_{NS}(\rho)$ is continuous. Then, upon finding a measure $\rho_0$ such that $c_{NS}(\rho_0) = 0$, we show that $c_{NS}(\rho)$ attains an interval $[0, c_0]$, for some (unknown) $c_0 > 0$, in particular $c_{NS}(\cdot)$ genuinely depends on the field. We then apply this theory on the particular case of toral eigenfunctions.
Hysteresis in immiscible fluid flow in porous materials

Adrian Sheppard,1,* Anna Herring,1 Zhe Li,1 Pieter Botha,1 Vanessa Robins,1 Olaf Delgado-Friedrichs,1 and Dorthe Wildenschild2

1Applied Mathematics, Research School of Physics and Engineering, The Australian National University
2School of Chemical, Biological and Environmental Engineering, Oregon State University

Geological materials, particularly sedimentary rocks, are excellent natural examples of random systems, with an original structure of deposited grains modified by subsequent processes of compaction, dissolution and precipitation. Many important processes involve the flow of two immiscible fluids through geologic materials: CO$_2$ and brine in an aquifer used for geologic CO$_2$ storage; water and air in a poorly consolidated soil of a hillside that may slip; oil, water and gas in petroleum reservoirs.

In general, such flows involve fluid dynamics, surface science and geometry. However, typical subsurface flows are at low velocities and take place in microscopic pore channels, leading to a flow regime where capillary forces dominate and the effects of viscous drag and turbulence can be neglected. In this regime, fluid invasion is a combination of slow, reversible creep and sudden, irreversible (dissipative) jumps from one equilibrium configuration to another. Capillary dominated flow can thus be thought of as a geometric process, arising from minimisation of the surface energy of the fluid-fluid and fluid-solid surfaces, each of which has a different surface tension. Fluid interfacial energy $\sigma_{12}$ leads to the Young-Laplace law, which relates the mean curvature of the interface $H$ to the pressure difference $\Delta P$ between the fluids on either side of it:

$$\Delta P = 2\sigma_{12}H. \quad (1)$$

At equilibrium, if both fluids remain hydraulically connected throughout the medium, then the pressure difference across the fluid interfaces will equal the capillary pressure $P_c$, the pressure difference applied between the inlet and outlet ports of the sample. Therefore the mean curvature of all fluid-fluid interfaces between connected fluid is the same at any moment in time.

Including the energy of the fluid-solid surfaces, $\sigma_{s1}$, $\sigma_{s2}$ leads to a well defined contact angle $\theta$ along the three phase contact line where the two fluids meet on the solid surface:

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\[ \theta = \cos^{-1}\left( \frac{\sigma_{s1} - \sigma_{s2}}{\sigma_{12}} \right) \]  

(2)

The symmetry between the fluids is broken by the differing fluid-solid surface energies (\( \sigma_{s1} \neq \sigma_{s2} \)); the fluid with a lower energy, usually water, is called the wetting fluid and tends to fill the smallest crevices of the pore structure. One of the most interesting aspects of two-phase fluid flow is that of hysteresis: the fluid configurations are different when water is being driven out (“drainage”) than when it is being drawn back in (“imbibition”). Hysteresis underpins capillary trapping, which is particularly important since it controls the residual oil that resists mobilisation from an oil field and the amount of CO\(_2\) that can be stored in a underground geological formation. This hysteresis is a major effect and is easily measurable in the most important physical quantities like capillary pressure, relative permeability and residual saturation. Despite being the subject of much study, no consensus yet exists on the principal mechanisms causing hysteresis in flow, or how macro-scale models may be constructed to take it into account. Part of the difficulty in understanding hysteresis is that multiple factors are implicated, including hysteresis in the contact angle itself, pinning of the interfaces at surface asperities and fluids choosing different flow paths.

Recent insights into these flows have come from 3D microscopy, particularly X-ray micro-tomography, that provides detailed images of the pore space structures down to the micron scale[1]. In addition, 4D tomography using synchrotron radiation can capture thousands of snapshots of the fluid configurations as one fluid displaces another from a porous sample. Results from 4D tomography suggest that localised high fluid velocities during interfacial jumps may have wider reaching implications than has been originally considered. However, there remain many interesting questions about equilibrium configurations. For a given pore element and a given interfacial mean curvature we would like to understand when more than one stable fluid configuration exists (“geometric bistability”), and the circumstances when one of these configurations becomes unstable, causing a sudden local rearrangement. If capillary equilibrium is assumed, then such a rearrangement causes a jump to another local configuration with the same mean curvature.

It is well understood that imbibition exhibits a richer set of possible rearrangements than drainage, due largely to the fact that a single imbibition event can involve the advance of wetting fluid from multiple crevices and constrictions. In drainage, all rearrangements can be assumed to be “Haines jumps”: sudden, piston-type advances from a single pore body through a constriction and into a wider region beyond. Haines jumps are well described by applying Mayer-Stowe-Princen (MS-P) theory to the two-dimensional surface of the constriction. In imbibition, invasion occurs by additional mechanisms, known as snap-off and co-operative pore filling, which arise when wetting menisci displace non-wetting fluid to the point that it disconnects from the surface. Reasonable approximations for this behaviour remain elusive in 3D for all but the simplest geometries.
Equilibrium fluid configurations can be studied numerically using the level set method [2] or by multi-component Navier-Stokes solvers. Such studies access different configurations by tuning the initial and boundary conditions, usually capillary pressure. Another option is to choose a more straightforward geometry, such as a packing of spheres, where many analytic solutions can be obtained [3]. However, any sort of complete picture remains elusive.

In this work we use X-ray micro-CT to capture fluid configurations at the pore scale in 3D, during drainage and imbibition cycles in a set of porous materials, including both unconsolidated grain packs and consolidated sandstones. Computation of topological invariants the Betti numbers $\beta_0$ and $\beta_1$ allows exploration of non-wetting (NW) fluid connectivity. $\beta_0$ results show that outright fluid disconnection plays a small role in hysteresis during drainage/imbibition cycles, whereas $\beta_1$ is strongly hysteretic, with the NW fluid showing an almost entirely dendritic structure during drainage of the unconsolidated samples. Given the likely importance of multiple flow pathways for fluid permeability, this is suggestive that NW fluid permeability will be lower during drainage than during imbibition and supports previous network model work showing that snap-off is suppressed in unconsolidated media [3, 4]. While less hysteresis is observed for the consolidated sandstones in our study, it is clear that snap-off is of minor importance for these systems as well.

Motivated by the spherical nature of typical fluid interfaces at pore-throat boundaries, as contrasted with the saddle-like curvatures of “pendular rings” that form around grain contacts, we also measure the Gaussian curvature $\kappa$ of the fluid interface. Strikingly, we find that $\kappa$ tends to change sign between drainage and imbibition, being on average positive during drainage, indicating spherical or ellipsoidal interfaces, but being negative during imbibition, indicating a dominance of saddle-like surfaces. Up until now, relatively little work has explored the role of the smaller principal curvature, but these results suggest that this may play a significant role in delaying the growth of pendular rings and suppressing snap-off.

Overall, our analysis of experimental data shows that connectivity and curvature...
analyses provide powerful, independent evidence that geometric bistability is at the heart of hysteresis in two-phase flow. Our results provide strong experimental support for existing modelling results [3, 4] which suggest that ’snap-off’ is almost nonexistent during imbibition in unconsolidated media, and plays a smaller then expected role in consolidated media.

Persistent homology signatures of spatially random systems

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One of the key objectives in studying apparently random point pattern data is to make quantitative statements about whether a given pattern is consistent with a particular theoretical model of the physical process that generated it. Stochastic geometry provides various tools to characterise random spatial patterns including functional summaries such as the 2-point correlation (or Ripley’s K) function and nearest-neighbour distance distribution, but it is known that more sensitive tests of structural difference are required in some circumstances. In [1], we showed how persistent homology can be adapted to standard statistical analyses such as hypothesis testing and functional principal component analysis (PCA) [1], and demonstrated its effectiveness in studying a variety of point patterns.

Persistent homology is an algebraic topological tool developed for data analysis that measures changes in topology of a filtration: a growing sequence of spaces indexed by a single real parameter. It produces invariants called the barcodes or persistence diagrams that are sets of intervals recording the birth and death parameter values of each homology class in the filtration. Statistical analysis of persistent homology has been difficult because the raw information (the persistence diagrams) are provided as sets of intervals rather than functions. Many research groups are pursuing various approaches to converting persistence diagrams to functional forms that are then amenable to standard statistical techniques. Our paper [1] uses the persistent rank functions which are analogous to cumulative distribution functions over the persistence diagrams.

For a point pattern, \( X \), we use the union of balls of radius \( a \) centred on the points, \( X_a = \bigcup_{x \in X} B(x, a) \) as the filtration with \( a \in [0, \text{diam}(X)] \). With this filtration, persistent homology captures the geometry of local multi-point configurations in an intuitively appealing way corresponding to connected components, loops or 1-dimensional holes and higher-dimensional cycles or cavities. In this setting, the rank functions generalise more familiar stochastic geometry summaries such as the nearest-neighbour (G) and empty space (F) functions. We test the power of persistent homology rank functions in discriminating three different 2D point process models from a complete spatially random null hypothesis in [1] and find that they do better overall than the standard functions mentioned above.
Another successful application of persistence diagrams is in the analysis of experimentally imaged configurations of approximately mono-disperse spherical bead packings. The persistence diagrams highlight the regular tetrahedral and regular octahedral configurations of crystalline sphere packing in an unambiguous way and have led to new insights in the grain-scale mechanisms underlying the order-disorder transition in this dissipative, athermal system, [2]. Functional PCA of the rank functions also reveals that the second-dimensional homology is more sensitive than the first-dimensional homology to the qualitative changes in structure observed at the Bernal packing fraction of 64%, see Fig. 1.

In terms of theoretical analysis, one drawback of homology rank functions is that they are not additive functionals, so are not amenable to calculations of the type possible for the Euler characteristic (another topological quantity) and other Minkowski functionals. On the other hand, it means that persistent homology may correlate with physical properties that are also not additive in the measure-theoretic sense. A natural example of this is the phenomenon of percolation. In [3] we present evidence of the connection between prominent length-scales in persistence diagrams and critical percolation length-scales in the purely random “swiss cheese” model and real sandstone porespaces imaged using micro-CT, see Fig. 2.
Plots of the separations between creator and destroyer critical points in persistence pairs are shown for a single sample in Figure 10. Again we see that the narrow peaks occur close to the relevant percolation thresholds, but the single maximal point can be significantly offset. A more sophisticated technique for determining the location of the persistence peaks would improve our estimates of percolation thresholds.

4.3. Mt Gambier Limestone

The Mt Gambier limestone sample is a counterpoint to the above sandstones and demonstrates the utility of persistence diagrams in giving a comprehensive picture of the pore space geometry and topology. A 2-D section and the persistence diagrams are shown in Figures 6c and 11; the distances between persistence pairs are summarized in Figure 12. This sample has no clearly defined length scales in the grain or

Fig. 2: Persistence diagram histograms for a Bentheimer sandstone core. The grey lines mark the radius of the maximal percolating sphere in the pore and grain phases.

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Applications of Minkowski Tensors in Gravitational Lensing

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1. Motivation and objectives
Minkowski tensors (MTs) have already been successfully applied in astrophysics and cosmology, e.g. to detect non-Gaussianities in the cosmic microwave background, [05], to compare simulated and observed galaxy distributions, [11], or to classify galaxies, [06]. Inspired by the results of [06], I discovered that they are also useful shape descriptors for the two-dimensional luminosity profiles of images generated from source galaxies by gravitational lenses. I will show that the MT decomposition of a luminosity profile efficiently distinguishes gravitationally lensed images from non-lensed objects and that the MT decomposition is more suitable to measure deviations from elliptical profile shapes than decomposing the profile into its central moments. Furthermore, I will introduce my MATLAB-Papaya-interface, which enables the use of the C++-implementation that calculates two-dimensional MTs, “Papaya”, in MATLAB.

2. A brief introduction to gravitational lensing
In analogy to optical lenses, gravitational lensing can be formulated in terms of a lensing equation

$$y = x - \alpha(x), \quad \alpha, x, y \in \mathbb{R}^2,$$

in which $y$ denotes a point in the source plane that is mapped onto a point $x$ in the image plane by adding a deflection $\alpha$ due to a mass distribution deflecting light rays emitted from the source, [07]. The deflection is assumed to be a function of the image plane position, projecting the entire lensing mass distribution along the line of sight into the image plane because the images are observed there, while the source is unobservably hidden behind the lens (see Fig. 1 for a visualisation of gravitational lensing). The lens usually consists of a luminous part, baryonic matter, and a dark matter

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part, so that the lensing mass distribution cannot be directly measured, either. Thus, the only observables in Eq. (1) are the luminosity profiles of the (multiple) gravitationally lensed images in the image plane, $I(x)$, so that inferring the luminosity profile of the source and the lensing mass distribution is a highly underdetermined problem.

Model-dependent descriptions of gravitationally lensed image configurations assume models for the luminosity profile of the source and the lensing mass distribution and adjust the models such that the luminosity profiles of the model-generated images match the observed ones. This approach requires tedious manual fine-tuning, it introduces a bias that may lead to incorrect astrophysical or cosmological conclusions, [04, 12], and, several models may describe the same image configuration equally well. To investigate the properties of an unobservable source and the lensing mass distribution – in particular the dark part – that can be inferred model-independently, I develop an approach that only relies on observable morphological features in the luminosity profiles of multiple images coming from one source, [08–10]. The reconstructions of the source and the lensing mass can be used to study galaxy evolution in the early universe and to better constrain the cosmological standard model.

3. Minkowski tensors as shape descriptors for luminosity profiles of lensed images

Image shapes are model-independently described by functional decompositions, [01–03]. MTs also belong to that class and decompose the luminosity profile into basis functions of decreasing degree of symmetry, similar to the central moments that are usually employed. While moments can be determined from noisy luminosity profiles directly, MTs require a segmentation along isocontours of constant luminosity first. However, MTs are more robust to noise and thresholding at different luminosities allows to inves-

![Fig. 1: As a massive object distorts space-time and deflects light rays (red curved rays), a telescope observation will not only show the bright foreground galaxy, but also arcs of light (red segments, “images”, right) of a background galaxy lying in the same line of sight as the foreground galaxy. Without having the foreground galaxy as “lens” the background galaxy would be unobservable. Its images are distorted and magnified due to the gravitational lensing effect.](image-url)
tigate deviations from self-similarity in the isocontours, given a sufficient resolution and signal-to-noise ratio.

Comparing the decomposition into moments with the MT decomposition, I found that deviations from elliptical profile shapes are detected more significantly using MTs, as shown in the example of Fig. 2. Measuring deviations from ellipticity is useful to characterise the lensing mass to greater detail, as it is linked to the moments (or MTs) and hierarchically decomposed on decreasing length scales.

Apart from the benefits of MTs to analyse deviations from elliptical shapes to infer properties of the lensing mass or reconstruct the source, I will also show that MTs can be used to detect highly-distorted images, so-called (giant) arcs. An example is shown in Fig. 3. I will show that the MTs related to the curvature, perimeter, area, and the length to width ratio of the arcs are able to distinguish arcs from non-lensed objects very efficiently, even from bright artifacts of stars and nearby luminous galaxies that have to be eliminated by additional postprocessing steps in other giant arc detection algorithms.

**Fig. 2:** Left: Simulated two-image-configuration. Centre: Two thresholds on the isocontours of both images, showing the decreasing resolution for increasing thresholds. Right: Deviation signal from an elliptical shape for image B from a moment-based approach (black line) and using Minkowski vectors (red line) as a function of the segmentation threshold.

**Fig. 3:** Left: Detail of a giant arc (red rectangle). Centre: Detection of this arc (red) using MTs with no spurious detections of large non-lensed galaxies. Right: Null-test, no arcs are detected in an arc-free field.
I thank Michael Klatt for very inspiring and helpful discussions and Sebastian Kapfer for his assistance to publish the MATLAB-Papaya-interface on http://theorie1.physik.uni-erlangen.de/research/papaya/. I gratefully acknowledge the support by the Deutsche Forschungsgemeinschaft (DFG) WA3547/1-1.

Adaption of the STIT tessellation model to crack structures – first steps

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The STIT tessellation model which was introduced in [1] can be a reference model for crack (or fissure) structures. In contrast to the Poisson hyperplane tessellation (PHT) and Poisson-Voronoi tessellation (PVT) it is a model where the cells are

Fig. 1: Simulations of Poisson-Voronoi, STIT, and Poisson line tessellations (from left to right).

Fig. 2: Craquelure (craquelé) on a ceramic surface

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Fig. 3: Simulation of a modified STIT tessellation where the distribution of the cell-dividing lines is more concentrated to the central part of the cells.

Fig. 4: Fracture trace map from geology (reproduced from [3]).

not face-to-face. Moreover, with respect to several aspects and quantities, STIT is 'between' PHT and PVT, see Fig. 1. However, already some tentative studies (e.g. for Fig. 2 the distribution of the size of cells) indicate that the STIT model often does not appropriately fit a real crack structure. Therefore an adaption based on a Gibbsian approach is worth studying. The theoretical background for that was provided in [2]. We suggest some specifications of this approach. For a tentative step see Fig. 3.

Furthermore, for an example from geology, shown in Fig. 4, we discuss another way to fit the model to data. Here, STIT tessellations with different directional distributions of the line segments are combined by iterating (also referred to as nesting) one into another one.

Mixing pears and oranges: Entropic self-assembly of bicontinuous and micellar phases

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The formation of nanoparticles and molecules into complex structures via self-assembly is an important aspect in many biological and synthetical systems. Lipids mixed with a solvent, for example, spontaneously form highly symmetric, cubic bilayer phases due to their amphiphilic nature [1]. However, it has been shown that next to those enthalpically driven processes also systems of hard particles, which only interact repulsively and thus are governed by entropy, arrange into larger macrostructures with long range translational or orientational order

![Diagram of phase diagram](image)

**Left:** Phase diagram of hard pear-shaped particles with length to width ratio $k = 3.0$ obtained by compression (from isotropic) and decompression at fixed tapering parameter $k_\theta$ for systems of 3040 particles in a cubic simulation box. Gray regions between the isotropic and ordered phases indicate parameter values for which phase hysteresis was observed between compression and decompression sequences. The schematics above the graph indicate the cross-sectional shape of the particles associated with each $k_\theta$ value. **Right:** Formation of inverse micelles by hard pear shaped particles in a pool of spheres. Only the pear particles are depicted.

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(crystallisation of hard spheres [2] and the nematic/smectic phases in systems of hard spherocylinders [3], respectively).

Here, we are particularly interested in a model of hard pear-shaped particles based on Gaussian overlap functions using Molecular Dynamics and Monte Carlo simulations, which forms – among other phases – a triply periodic, bicontinuous minimal surface structure - the $Ia3d$ double gyroid [4]. Studying its geometrical and morphological properties, we provide the complete phase diagram of this system, with global density and particle shape as the two variable parameters [5]. Additionally, we probe the mechanism by which interdigitating sheets of pears in these systems create surfaces with negative Gauss curvature, which is needed to form the Gyroid minimal surface.

To model mixtures similar to a lipid-water system hard spheres, which take up the role of the solvent, are introduced into the simulations. By analysing configurations of different concentrations and sphere sizes, we can investigate mechanisms which potentially enable us to stabilise metastable phases like the $Pn3m$ double diamond within the mixture, but can also observe clusters of pears reminiscent of inverse micelles. From this we expect to get a deeper understanding in the formation of bicontinuous structures in nature and a notion of the similarities and differences between enthalpic and entropic self-assembly.

Tuesday 12 September 2017

chair: Myfanwy Evans

09:00 – 09:50  **KN Krauth**  
*Fast irreversible Markov chains and their applications in statistical physics*

09:50 – 10:30  **IT Drenckhan / Giustiniani**  
*Hard science with soft spheres: learning from foams and emulsions*

**coffee break**

chair: Werner Krauth

11:00 – 11:40  **IT Coerjolly**  
*Lennard-Jones potential estimation*

11:40 – 12:20  **IT Evans**  
*Heterogeneous forces in squashed foams*

**lunch**

chair: Michael Klatt

14:30 – 15:20  **KN Steinhardt**  
*Hyperuniformity of Quasicrystals and Related Patterns*

15:20 – 16:00  **IT Calka**  
*The typical Poisson-Voronoi cell around an isolated nucleus*

**coffee break**

chair: Daniel Hug

16:30 – 17:00  **CT Hutzler**  
*Bubble-bubble interactions in 2D, just above the jamming transition*

17:00 – 17:30  **CT Nedelec**  
*A theory that predicts behaviors of disordered cytoskeletal networks*

17:30 – 18:00  **CT Heydenreich**  
*Scale-free percolation*

**Dinner**

**20:00 – 22:00 Poster session**
Fast irreversible Markov chains and their applications in statistical physics

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The Monte Carlo method is an outstanding computational tool in science. Since its origins[1], it has relied on the detailed-balance condition, i.e. on the concept of reversible Markov chains, to solve general computational problems under the conditions of thermodynamic equilibrium with zero probability flows.

In this talk, I discuss irreversible Markov chains that violate detailed balance, yet satisfy global balance[2, 3]. Equilibrium is reached as a steady state with non-vanishing probability flows. For one-dimensional particle models, one may show explicitly that these algorithms reach equilibrium on much faster time scales than the reversible algorithms[4]. The generalization of irreversible Markov chains to higher dimensions succeeds if the Metropolis acceptance criterion based on the change in the energy is replaced by a new consensus rule, the factorized Metropolis algorithm[3]. The system energy is not computed, providing a fresh

Event-chain algorithm[2], an early example of irreversible Markov chains with infinitesimal moves. The algorithm generalizes to arbitrary interactions via a “consensus” principle[3]. It has been instrumental to the recent solution of melting problems in two dimensions[7, 8].

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perspective for long-range interactions[5]. Moves are infinitesimal and persistent, and employ the powerful lifting framework[6].

As an application I discuss our recent solution of the 2-d melting problem for hard disks[7] and general potentials[8] that relies on the new class of powerful Markov chains, and specifically on the event-chain algorithms[2, 3]. These algorithms have allowed us to demonstrate the existence of two phase transitions, from a liquid to a hexatic and to a solid phase. The liquid-hexatic transition is first order for steep potentials (including hard disks), and of Kosterlitz-Thouless type if the inter-particle interactions are sufficiently smooth.

Hard science with soft spheres: learning from foams and emulsions

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Who has not marvelled at the delicate organisation of bubbles floating on top of a beer or a bathtub? Or similarly, of oil drops hovering on the surface of a soup or a vinaigrette? Under gravity, those soft and near-frictionless bubbles and drops snuggle neatly together, creating sphere-type packings where the influence of gravity is small, and polyhedral packings, where the influence of gravity is important.

Being able to understand and to predict these packings has challenged physicists and mathematicians alike; and their close collaboration has brought forward an increasingly deep understanding of the intriguing features that characterise the packing procedures and the final structure of foams and emulsions.

Building on these advances, physical chemists are now joining this subject. The
necessity of stabilising foams or emulsions with interfacially active agents (soap-like molecules, block-copolymers, particles,...), provides a vast playground to tune (and to understand) the interactions of these soft spheres via specific modifications of their surfaces: How do the interactions change if the bubbles/drops are covered by a visco-elastic skin? How do they pack if this skin creates non-negligible friction? And what if these spheres are adhesive?

In this presentation we will give an overview of the state of the art of this interdisciplinary field in bridging historical aspects with recent findings.
Lennard-Jones potential estimation

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Among models of spatial point processes in $\mathbb{R}^d$, the class of (stationary) pairwise interaction point processes is probably the most well-known due to its simple mechanistic interpretation. Briefly, such a Gibbs point process is defined (in a bounded domain) by its density with respect to a Poisson process with unit rate, given by

$$f(x) \propto \beta \exp \left( - \sum_{u,v \in x} \Phi(u-v) \right)$$

for any configuration of points $x$. The function $\Phi$ is naturally called the pairwise interaction function and $\beta$ is a real parameter. Alternatively such models are also described (and actually characterized) by their Papangelou conditional intensity given for any $u \in \mathbb{R}^d$ and any configuration of points $x$ by

$$\lambda(u, x) = \beta \exp \left( - \sum_{v \in x} \Phi(u-v) \right).$$

The function $\Phi$ is assumed, for simplicity, to be isotropic, and is still denoted by $\Phi$.

Among them, the Lennard-Jones model plays a historical and central role. Introduced by physicists in the early 1900’s (see e.g. [1]), this model, also known as the 12-6 potential, has pairwise interaction function

$$\Phi_{\text{LJ}}(r) = \frac{\theta_1}{r^{12}} - \frac{\theta_2}{r^6}.$$ 

Its main characteristics are the non-boundedness at small distances and its infinite range. To compare with, the Strauss model and Strauss Hard-Core model are pairwise interaction point processes with pairwise interaction function with finite range $R$ (i.e. $\Phi(r) = 0$ for $r \geq R$) respectively given by

$$\Phi_{\text{Strauss}}(r) = \theta 1(r \leq R), \quad \theta \geq 0$$

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and
\[ \Phi_{\text{STRAUSS HC}}(r) = (+\infty) \mathbf{1}(r < \delta) + \theta \mathbf{1}(\delta \leq r \leq R), \quad \theta \in \mathbb{R}. \]

The latter model means that no configuration with two points at distance smaller than \( \delta \) is allowed. Fig. 1 these three functions.

Many probabilistic properties of Lennard-Jones type models have been established by Ruelle in the 70’s, see [2] (including the existence of such models). However from a statistical and practical point of view, the infinite range of the process poses additional problems, which had never been considered.

The pseudolikelihood, [3], and logistic regression methods, [4], are statistical procedures to estimate a parametric Papangelou condition intensity. These methods are efficient alternatives to the likelihood, a method almost never considered for Gibbs models due to its complexity (from a computational as well as from a theoretical aspect). The pseudolikelihood and logistic regression methods can be efficiently implemented and are proven to be asymptotically consistent for most of models having a finite range (which includes Strauss, Strauss Hard-Core models among others).

In collaboration with F. Lavancier, we have proposed extensions of the pseudolikelihood and the logistic regression method designed to take into account the infinite range of this process and proved asymptotic properties for the corresponding estimators. In particular, within the increasing domain setting, we have proved that the procedures are consistent and that the estimators are asymptotically normal.

When the pairwise interaction function has a finite range, the main tool for deriving asymptotic results is a central limit theorem for conditionally centered Markov random fields. This kind of CLT is perfectly well-suited for Gibbs models since, first, they do not require any mixing-type condition, a condition which is unknown even for simple models, and second, it can be applied even if the Gibbs measure is not unique.

The main contribution of our work is to extend this general central limit theorem to triangular arrays of random fields which are almost conditionally centered.

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**Fig. 1:** Pairwise interaction function for a Strauss model, a Strauss Hard-core model and a Lennard-Jones model.
This result combined with the infinite range characteristic of the model impose some restrictions on the pairwise interaction function. These conditions are however satisfied for the Lennard-Jones model. A simulation study will show that the method works well for the Lennard-Jones model. The reader interested in the details is referred to [5].

Heterogeneous forces in squashed foams

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We explore an alternate way of looking at the rheological response of a yield stress fluid: using discrete geometry to probe the heterogeneous distribution of stress in soap froth. We present quasi-static, uniaxial, isochoric compression and extension of three-dimensional random monodisperse soap froth in periodic boundary conditions and explore quantitative morphometric tools to analyse the distribution of cell deformation and cell stress within deformed foams. Cumulatively, the spatial distribution of highly deformed cells allows us to examine how stress is internally distributed [1].

We can conclude that the highly deformed cells become more correlated in space as the foam is deformed (be it by compression or by extension). We use the notion of looking for spatial correlations via deviations from topological invariants for random spatial patterns. We suggest that such heterogeneous correlations of highly deformed cells throughout the deformed foam starts to build a notion

Compression simulations of a monodisperse foam with 512 cells, shown at 3 different strains from left to right, with cells coloured by a measure of deformation. (Top) The most deformed 256 cells are shown at each strain. (Bottom) The 50 most highly deformed cells are shown

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equivalent to force-chains in granular material, and that such methods could be also applied to force networks in granular materials as well as other disordered media. Further, we suggest that using the extrema of the $\chi$-distribution to dictate the thresholding of forces when searching for force chains in granular material adds a new methodology to the various existing thresholding techniques.

Some recent studies of quasicrystals and related patterns have revealed a remarkable range in the degree of hyperuniformity and, in some cases, the absence of hyperuniformity altogether [1–3]. This talk will discuss these results and some initial studies of their physical implications [4].

PJS gratefully acknowledges the many contributions of his collaborators, C. Lin, E. Orguz, J. Socolar, and S. Torquato, whose work is reviewed in this...
presentation.

The typical Poisson-Voronoi cell around an isolated nucleus

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We aim at describing Poisson-Voronoi configurations in regions low in nuclei. To do so, we fix a convex body $K$ containing the origin $o$ and we construct the planar Voronoi tessellation generated by the union of the origin and a homogeneous Poisson point process of intensity $\lambda$ conditioned on the event that the cell associated with $o$ contains $K$. The Voronoi cell $K_\lambda$ of $o$ is then an approximation from the outside of $K$, see Fig. 1.

![Fig. 1: On the left: the convex body $K$ around the origin $o$ (red), its Voronoi flower (blue) and the Poisson point process $P_\lambda$ conditioned on $K$ being included in the Voronoi cell of $o$ (black); on the right: the resulting Voronoi cell $K_\lambda$ associated with the origin (red).]

When the intensity of the Poisson point process goes to infinity, we obtain explicit asymptotics for the means of the area $A(K_\lambda)$, perimeter $U(K_\lambda)$ and number of vertices $N(K_\lambda)$ of $K_\lambda$. As in Rényi and Sulanke’s seminal papers on random convex hulls [4], the regularity of the boundary of the convex body $K$ is of crucial importance. In particular, when the boundary $\partial K$ of $K$ is of class $C^2$

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with bounded positive curvature, the three expectations satisfy

\[
E(\mathcal{A}(K_\lambda)) - \mathcal{A}(K) \sim \lambda^{-\frac{2}{3}} 2^{-\frac{2}{3}} 3^{-\frac{1}{3}} \Gamma\left(\frac{2}{3}\right) \int_{\partial K} r_s^\frac{2}{3} \langle s, n_s \rangle^{-\frac{2}{3}} ds
\]

\[
E(\mathcal{U}(K_\lambda)) - \mathcal{U}(K) \sim \lambda^{-\frac{2}{3}} 3^{-\frac{4}{3}} \Gamma\left(\frac{2}{3}\right) \int_{\partial K} r_s^{-\frac{2}{3}} \langle s, n_s \rangle^{-\frac{2}{3}} ds
\]

\[
E(\mathcal{N}(K_\lambda)) \sim \lambda^{\frac{1}{3}} 2^{\frac{2}{3}} 3^{-\frac{4}{3}} \Gamma\left(\frac{2}{3}\right) \int_{\partial K} r_s^{\frac{2}{3}} \langle s, n_s \rangle^\frac{1}{3} ds
\]

where \( r_s \) and \( n_s \) are respectively the radius of curvature and the outer unit normal vector of \( \partial K \) at point \( s \).

When \( K \) is itself a polygon with \( n_K \) vertices, the behavior of these means becomes

\[
E(\mathcal{A}(K_\lambda)) - \mathcal{A}(K) \sim \lambda^{-\frac{1}{2}} 2^{-\frac{3}{2}} \pi^2 \sum_{i=1}^{n_K} ||o_i||^{-\frac{1}{2}} ||a_{i+1} - a_i||^\frac{3}{2}
\]

\[
E(\mathcal{U}(K_\lambda)) - \mathcal{U}(K) \sim (\lambda^{-1} \log \lambda) \cdot 2^{-1} 3^{-1} \sum_{i=1}^{n_K} ||o_i||^{-1}
\]

\[
E(\mathcal{N}(K_\lambda)) \sim (\log \lambda) \cdot 2 \cdot 3^{-1} n_K
\]

where \( a_1, \ldots, a_{n_K} \) are the consecutive vertices of \( K \) and \( o_i \) is the projection of \( o \) onto the line \((a_i, a_{i+1})\) with the convention \( a_{n_K+1} = a_1 \). All these estimates are based notably on accurate estimates of the area of the Voronoi flower and of the support function of \( K_\lambda \) as well as on an Efron-type relation. They extend the results contained in [1] in the particular case when \( K \) is a disk. Estimating the quality of a random polyhedral approximation from the outside was also at the heart of a recent paper by D. Hug and R. Schneider which provides an estimate of the Hausdorff distance between \( K \) and a slightly different model of a zero-cell from a stationary Poisson hyperplane tessellation [3].

In a second step, we turn our attention to the asymptotic shape of two other random polygons:

- the Voronoi cell \( \hat{K}_\lambda \) containing \( K \) when the Poisson point process is conditioned on the event that \( K \) is included in one of the Voronoi cells;
- the Voronoi cell \( \mathcal{C}_\lambda(D) \) associated with the origin when the point process is conditioned on not intersecting a fixed bounded closed set \( D \) containing \( o \) in its interior, see Fig. 2.

We show in particular that the nucleus of \( \hat{K}_\lambda \) converges in distribution to the Steiner point of \( K \) and that \( \mathcal{C}_\lambda(D) \) converges almost surely in the Hausdorff metric to a specific deterministic convex body \( K \) bounded by the antiorthotomic curve of \( D \). We deduce from these results the asymptotic means of their area, perimeter and number of vertices.
Fig. 2: On both sides: the domain $D$ around the origin $o$ (blue), the Poisson point process outside $D$ (black), the associated Voronoi cell $C_\lambda(D)$ of the origin (red).

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Bubble-bubble interactions in 2D, just above the jamming transition

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Two-dimensional wet foams or weekly compressed emulsions are often modelled as circular disks whose overlap is resisted by a force that varies linearly with distance. Despite the insight that this so-called bubble model [1] has provided for foam rheology (when augmented by a viscous dissipation term) [2, 3], it is unrealistic in various respects. In particular Morse and Witten [4] have shown that drops/bubbles just above the jamming transition do not obey simple pairwise force laws. On the other hand, numerical simulations based on balancing surface tension and pressure force (2D software Plat [5, 6]) or minimising interface energy (Surface Evolver [7]), which work well for dry foams, have limited success for wet foams.

Here we present the development of Morse and Witten’s approach to relative forces and positions for wet foams in a scheme analogous to that of Höhler and Cohen-Addad in 3D [8]. We hope to clarify the discrepancy that has recently arisen between simulations and experiments.

2D wet foam (packing fraction $\phi = 0.88$) as represented in (a) the bubble model, (b) a Plat simulation, and (c) the force model described here.

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emerged in the variation of average contact number of bubbles with liquid frac-
tion, namely a square root variation in the bubble model, and a linear relation in Plat simulations which consider adjustments of bubble shapes [10]. based on the Morse-Witten formalism.

The Morse-Witten force law deserves to take its place as one of the canonical laws of soft matter, but has hitherto been poorly appreciated, largely on the account of the difficulty of interpreting the original paper. Our re-formulation of the theory, initially in 2d, should offer clarification.

ACKNOWLEDGMENTS

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Morphogenesis in animal tissues is mainly driven by tensions created in the cortex of cells, a random network of actin filaments crosslinked by dedicated proteins such as myosin motors. Within the cortex, the motion of the molecular motors powered by ATP generates an active contractile force resulting in overall build up of tension. Although the network components and their properties are known, the detailed mechanism giving rise to this contractility is still poorly understood.

Guided by many simulations such as this one, we developed an analytical theory that predicts the contractile rate of a network from its principal characteristics.
understood. We will describe a theory that predicts whether an isotropic random network will contract, expand, or conserve its dimensions, depending on the properties of the filaments and the elements that connect them. This analytical theory correctly predicts the behaviour of simulated networks consisting of filaments with varying combinations of connectors, and reveals conditions under which networks of rigid filaments are either contractile or extensile. Our results suggest that contractile networks in which filaments turn over are intrinsically pulsatile. The theory encompasses mechanisms of contractions previously proposed. It provides a foundation for the study of a broad range of processes involving cytoskeletal networks, and a basis for designing synthetic networks.

[1] A Theory That Predicts Behaviors Of Disordered Cytoskeletal Networks
Scale-free percolation

Markus Heydenreich*

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Many real-world networks, such as WWW, social, financial, neural, or biological networks, exhibit a number of fairly general patterns:

- the length of a smallest path between two vertices is small w.r.t. the system size (‘small world’)
- the degrees of vertices exhibit a power law (a ‘scale-free network’)
- vertices that are geographically close are likely to be connected (‘geometric clustering’)
- vertices with high degree are likely to be connected even if far away from each other (‘hierarchies’)

It is a challenge to find good mathematical network models that are rich enough to capture these properties but sufficiently simple to be amenable to a rigorous analysis. Scale-free percolation, as introduced by Deijfen, van der Hofstad, and Hooghiemstra (2013), is an excellent candidate that meets all of the above criteria. It denotes a percolation model on $\mathbb{Z}^d$, where two points $x$ and $y$ are connected by an edge with probability

$$1 - \exp\left(-pW_xW_y/\text{dist}(x,y)^b\right)$$

where $p>0$ is a percolation parameter, $W_x$ and $W_y$ are i.i.d. edge weights with power law distribution, and $b$ denotes the exponent for the long-range connections. It arises as a marriage of the (non-spatial) Norros-Reittu model and the (non scale-free) long-range percolation. Bringmann, Keusch, and Lengler (2016) consider a variant of this model in finite domain and continuous space. Indeed, they prove that for a certain parameter range the main results from the model on infinite lattice remains valid. After an introduction to the model, I shall focus on the regime where the expected degrees are infinite. I present criteria for

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transience vs. recurrence, and the emergence of a specific hierarchical network as subgraph of the infinite cluster.

The talk is based on joint work with Joost Jorritsma and Tim Hulshof.
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Metastability of the contact process on fast evolving scale-free networks

Emmanuel Jacob,\textsuperscript{1} Amitai Linker,\textsuperscript{2} and Peter Mörters\textsuperscript{3,}\textsuperscript{*}

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\textsuperscript{2}Universidad de Chile
\textsuperscript{3}University of Bath and Universität zu Köln

In this talk we investigate the effect of fast network dynamics on the behaviour of the contact process (see Fig. 1) on scale-free networks modelled as inhomogeneous random graphs with suitable connection kernels. The stationary network dynamics consists of vertices updating their neighbourhoods independently of the contact process. Variation of a parameter $\eta$, which controls the rate at which the most powerful vertices update, allows an interpolation between a scenario where vertices update on the time-scale of the contact process and a mean-field model where updates occur on a time scale of much faster order.

\textit{Fig1: In the contact process every vertex of a graph can be either healthy or infected. Infected vertices (denoted by filled circles) recover at rate one, healthy vertices (open circles) get infected at a rate given by the number of infected neighbours multiplied with an infection rate $\lambda > 0$. In the evolving network every vertex additionally updates its neighbourhood at a rate $\kappa > 0$ depending on the vertex strength.}

We study the behaviour of the extinction time of the infection. We say there is \textit{fast extinction} if, for some sufficiently small infection rate $\lambda > 0$, the expected extinction time is bounded by a power of the network size, and there is \textit{slow extinction} if, for every infection rate $\lambda > 0$, the expected extinction time is at least exponential in the network size with high probability. Slow extinction is a phenomenon of \textit{metastability}, a physical system reaching its equilibrium very slowly because it spends a lot of time in states which are local energy minima, the metastable states. In our model the metastable states are characterised by a density of infected vertices, the \textit{metastable density}. Different survival mechanisms for the infection lead to different exponents describing the decay of metastable densities as $\lambda \downarrow 0$, and phase transitions in these exponents are strongly indicative

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of changes in the optimal survival mechanism of the infection when the network parameters vary.

The focus of this talk is on two paradigmatic connection kernels, the factor kernel and the preferential attachment kernel, which exhibit very different behaviour. For the factor kernel we identify a phase transition between fast and slow extinction, and, in case $\eta < \frac{1}{2}$, a further transition in the behaviour of the metastable densities within the slow extinction phase. For the preferential attachment kernel a phase transition between fast and slow extinction only occurs when $\eta > \frac{1}{2}$. For $\eta < \frac{1}{2}$ we always have slow extinction and two phase transitions in the behaviour of the metastable densities. These results are summarised in Fig. 2.

**Fig2:** Phase diagram with fast extinction phase in blue, other phases labelled with the corresponding optimal survival mechanism. The parameter on the horizontal axis is $\gamma \in (0, 1)$, which describes how the mean degree grows with the vertex strength (related to power-law exponent by $\tau = 1 + \frac{1}{\gamma}$), and the parameter on the vertical axis is $\eta \in [0, \infty)$ which controls the rate at which the strongest vertices update.
Branched Covering Surfaces

Konrad Polthier

Freie Universität Berlin

The boundaries between virtual and physical geometrical forms dissolve, allowing for new intersections between the two worlds. New 3D Scanners offer insights on the most diverse size scales and reveal an ever-increasing variety of geometrical forms in nature. New 3D printing technologies entail a manifold of physical properties and production processes. Until now, only little is understood, the mathematical description language for the various forms exists only rudimentarily, just as there are only few effective modeling processes. Starting from historical geometries and their issues, and with the aid of selected examples from biology computer graphics and industrial CAD, the talk discusses open questions regarding the new wealth of geometric forms.

As a demonstrating example, the issue of multivalued functions will be introduced and resolved using branched covering surfaces, followed by applications in several key problems. Multivalued functions and differential forms naturally lead to the concept of covering surfaces and more generally of covering manifolds. We will illustrate and discretize basic concepts of branched covering surfaces starting from complex analysis, surface theory up to their recent appearance in geometry processing algorithms. Applications will touch artistic surface modeling, geometry retargeting, surface and volume parameterization, and novel weaved surface representations.
A random walk through the cosmic web

Ravi K. Sheth*

Center for Particle Cosmology, Department of Physics and Astronomy, University of Pennsylvania

The Cold Dark Matter model provides a rather successful description of the observed clumps, filaments and voids in the cosmic web. The 'cold' is important: it allows one to connect the structures we observe today to the initial conditions about 13 billion years ago from which they formed. I will review why. In particular, I will describe a simple approach which maps many questions about cosmological structure formation into the language of random walks first crossing suitably chosen barriers. Whereas early work was based on the idealization that the walks are Markovian and Gaussian – where there are nice connections between the random walk picture and coagulation-based approaches – there is recent interest, and many open questions, in the case where they are neither.

Inferred velocity flows and the observed cosmic web (from Wang et al. 2012).

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Left: Schematic drawing of the initial spatial distribution of objects that gives rise to the merger history tree shown on the right. The largest circle represents the comoving size of the initial region associated with the final collapsed bound 'halo'. As time evolves this comoving radius decreases. The assumption is that all the matter initially within this region remains within it always. Thus, information about how the mass of a final object was partitioned into subhaloes at a given time contains information about the halo distribution smoothed on a scale given by the radius of the larger object at that time. Right: Schematic drawing of the associated merger history tree. Time increases upwards: the initial time is at the bottom of the figure. The branch on the right is associated with a region that, initially, was made up of many small objects that were close to each other, but rather separated from any other objects. The branch on the left, on the other hand, is associated with a region that was initially populated rather more homogenously.

Random hyperplane tessellations: their polytopes and cones

Rolf Schneider*

University of Freiburg

Random mosaics generated by random hyperplane processes in Euclidean space are a much studied object of Stochastic Geometry, and their typical cells or zero cells belong to the most prominent models of random polytopes. We consider either stationary Poisson hyperplane processes in $\mathbb{R}^d$, which tessellate the space into convex polytopes, or finitely many independent random hyperplanes through the origin, which divide the space into finitely many polyhedral cones. Three topics concerning the generated random polytopes and cones will be selected. The first topic, presenting joint work with Matthias Reitzner [3], asks for the possible shapes of the polytopes in a stationary Poisson random mosaic. With probability one, they are simple polytopes, but what else can be said? The second topic deals with first and second moments of some geometric functionals, such as face numbers or volumes of skeleta, and concerns both models. For stationary Poisson hyperplane tessellations, second moments can be determined, for example, for the vertex number of the typical cell, and an extremal property of the isotropic case is obtained ([4]). For typical random cones generated by a finite number of independent isotropic random hyperplanes through the origin, some first moments were already calculated by Cover and Efron [1]. In joint work with Daniel Hug [2], this was continued, and some second moments were found. The third topic is concerned with the same random cones, and we determine some intersection probabilities with fixed or moving cones ([5]).


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A probabilistic view on high dimensional convex bodies

Christoph Thäle*

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In the first part of this talk we will summarize a number of (by now classical) results about the geometry of high-dimensional convex bodies. Especially, we shall review the large deviation inequality, the thin-shell property and the central limit theorem. In the second part of the talk we present a number of recent results for a particularly interesting class of convex bodies, the $\ell_p$-balls. In particular, we discuss new (quantitative) central limit theorems and large deviation principles together with selected applications.
Random convex-valued processes and some general iterative schemes

Ilya Molchanov*

University of Bern

The talk recalls known models of random convex bodies, describes several new models, and then continues to discuss set-valued processes whose realisations are random convex bodies. The ideas of set-valued martingales and autoregressive processes will be briefly recalled. The main emphasis will be put on processes (set-valued or conventional ones), which arise from sieving iterative function systems of a very general nature.

The talk is based on joint work with Alexander Marynych (Kiev University).

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Metastability for continuum interacting particle systems

Frank den Hollander\textsuperscript{1} and Sabine Jansen\textsuperscript{2,*}

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\textsuperscript{2}University of Sussex, UK

We consider a system of point particles in a finite box in $\mathbb{R}^2$ interact via a finite-range attractive pair potential and move according to a Markov process that has the grandcanonical Gibbs measure as a reversible measure. The pair potential has a hard core, compact support, and a unique minimum; the minimizer of the energy is a triangular lattice. The dynamics is a birth and death process, without particle hopping, which is a continuum version of the Glauber dynamics from spin systems.

The chemical potential is such that the system favors a packed box in a crystalline ground state, but has a nucleation barrier to overcome in order to go from an empty box to a filled box. We are interested in the nucleation time in the limit as the temperature tends to zero, at fixed chemical potential and fixed volume. We use the potential-theoretic approach to metastability to prove an Arrhenius law. The results extend earlier work for lattice systems.

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Convex Hulls of Random Walks: Expected Number of Faces

Zakhar Kabluchko,\textsuperscript{1,*} Vladislav Vysotsky,\textsuperscript{2} and Dmitry Zaporozhets\textsuperscript{3}

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\textsuperscript{2}University of Sussex and St. Petersburg Department of Steklov Mathematical Institute
\textsuperscript{3}St. Petersburg Department of Steklov Mathematical Institute

Consider a random walk $S_i = \xi_1 + \cdots + \xi_i$, $1 \leq i \leq n$, starting at $S_0 = 0$, whose increments $\xi_1, \ldots, \xi_n$ are random vectors in $\mathbb{R}^d$, $d \leq n$. We are interested in the properties of the convex hull $C_n := \text{Conv}(S_0, S_1, \ldots, S_n)$. Assuming that the tuple $(\xi_1, \ldots, \xi_n)$ is exchangeable and some general position condition holds, we prove that the expected number of $k$-dimensional faces of $C_n$ is given by the formula

$$
\mathbb{E}[f_k(C_n)] = \frac{2 \cdot k!}{n!} \sum_{l=0}^{\infty} \left[ \frac{n+1}{d-2l} \right] \left\{ \frac{d-2l}{k+1} \right\},
$$

for all $0 \leq k \leq d-1$, where $\left[ \frac{n}{m} \right]$ and $\left\{ \frac{n}{m} \right\}$ are Stirling numbers of the first and second kind, respectively. Generalizing the classical discrete arcsine law for the position of the maximum due to E. Sparre Andersen, [On fluctuations of the sums of random variables, I: Math. Scand. 1 (1953) and II: Math. Scand. 2 (1954)], we compute explicitly the probability that for given indices $0 \leq i_1 < \cdots < i_{k+1} \leq n$, the points $S_{i_1}, \ldots, S_{i_{k+1}}$ form a $k$-dimensional face of $\text{Conv}(S_0, S_1, \ldots, S_n)$. This is done in two different settings: for random walks with symmetrically exchangeable increments and for random bridges with exchangeable increments. The main ingredient in the proof is the computation of the probability that the origin is absorbed by a joint convex hull of several random walks and bridges whose increments are invariant with respect to the action of a direct product of finitely many reflection groups of types $A_{n-1}$ and $B_n$. This probability, in turn, is related to the number of Weyl chambers of a product-type reflection group that are intersected by a linear subspace in general position. All formulae are distribution-free, that is do not depend on the distribution of the $\xi_k$’s.


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Nodal statistics of random plane waves

Ivan Nourdin,* Giovanni Peccati, and Maurizia Rossi

University of Luxembourg

According to Berry’s conjecture, the local behavior of high-energy wavefunctions in classically chaotic quantum billiards can be predicted by that of random plane waves.

In this talk we investigate the distribution of nodal statistics for Berry’s Random Wave Model. We focus in particular on the length of nodal lines and the number of nodal points in a compact set, and we study their fluctuations as the domain diverges to the whole plane. The latter turn out to be Gaussian, in contrast with the case of the standard flat torus where noncentral limit theorems have been recently obtained.

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Goodness-of-fit tests for complete spatial randomness based on Minkowski functionals of binary images

Bruno Ebner,¹,∗ Norbert Henze,¹ Michael A. Klatt,¹,² and Klaus Mecke²

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²Institute of Theoretical Physics 1, FAU Erlangen-Nürnberg

Serious analysis of spatial point patterns always start with a goodness-of-fit test to detect departure from the absence of structure in the data. Whether one has a known or a random number of points two different models apply, namely the uniform distribution (for goodness-of-fit of uniformity, see [2]) and the concept of complete spatial randomness, which is synonymous for a homogeneous Poisson point process. We propose for the latter case some new goodness-of-fit procedures for data in a given observation window. Therefore we use an equidistant binning of the observation window and apply a threshold procedure, see Fig. 1, to generate binary images which are then analyzed by Minkowski functionals like the area, the perimeter and the Euler characteristic. Examples of the transition to a binary image can be found in Fig. 2. We demonstrate that the computation of Minkowski functionals of binary images is efficient by utilizing techniques of image analysis resulting in a look-up table.

Asymptotic distributional theory is derived under the hypothesis and a suitable limiting regime using a limit theorem for dependency graphs, incorporating the local dependency structure and representations as sums of random variables of

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the Minkowski functionals. Under some specific fixed alternative inhomogeneous Poisson point processes, we show almost sure limits leading to first consistency statements. We compare the new methods to existing procedures (as presented in [1]) in a Monte Carlo simulation study. Finally we apply the new tests to a real data set, namely the gamma-ray sky map of the Fermi Gamma-ray Space Telescope, where we analyze a sky-map of gamma-rays observed by the so-called Large Area Telescope, see Fig. 2 for pictures of the investigated data sets.

Chemical solutions are objects for GPSRS study

Nikolai N. Medvedev*

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The structure of aqueous solutions of small amphiphilic molecules is a subject of many researches in chemistry and molecular biology. Solute molecules can be distributed homogeneously or begin clustering at the low concentration, or can form an inhomogeneous “micro-phases” with the solvent molecules. Traditionally, chemists study solvation shells of the solute molecules. They represent a local order of the solution, and directly reflect intermolecular interaction between the solute and solvent, which may be studied experimentally. The local intermolecular interaction together with the space-filling laws generate a medium range order of a solution, and ultimately a global spatial arrangement of the solute molecules.

From geometrical point of view, liquid solutions are typical spatial random systems. However, the methods for structural analysis of solutions, known in chemistry, are not sufficient. The chemists did not tackle to development of the methods because there were no representative computer models of the solutions on molecular level. To get such models needs intensive molecular dynamics simulation. Only in the last decades, it became possible to calculate models to study entire structure of solutions. The methods developed for analysis of spatial random systems in different fields of researches can be applied also to solutions.

We have generated a set of molecular dynamics models of aqueous solutions of some amphiphilic molecules, which are interesting for biology. The structure of the solutions at different concentration was investigated by the Voronoï-Delaunay method and cluster analysis. The spatial arrangement of the solute molecules is compared with the arrangement of randomly distributed hard spheres at different packing fraction (from 0 to 0.2), [1]. Voronoi cell volume distribution of random hard spheres is shown in Fig. 1. It is rather broad, like for random points (dashed line), but the variance of the distribution decrease with increase of packing fraction, see Fig. 2, black line.

It was obtained that the molecules of trimethylamine-N-oxide (TMAO) are distributed in the solutions as the random spheres at the corresponding concentration: their curves of the Voronoï cell volume variance vs. packing fraction coincide very well, see red symbols and black line, Fig. 2. This is an unexpected result,

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because in contrast to hard spheres, solute molecules have a specific interaction between each other and the solvent (water, in our case). Indeed, the curves for other molecules, tert-butyl alcohol (TBA) and urea, demonstrate different behavior.

Larger value of the variances for TBA indicates inhomogeneity of the solutions. Standard cluster analysis shows strong increase of the mean cluster size of TBA molecules with concentration, Fig. 3. TMAO clusters, as one would expect, grow up as in system of random spheres.

A more detailed analysis of the clusters can be performed with Delaunay simplexes. If all the vertices of a Delaunay simplex are defined by atoms of the solute molecules, then it represents an internal part of a cluster of the molecules. A simplex formed only by water molecules belongs, obviously, to a water region. A simplex whose vertices are composed of both water and atoms of dissolved molecules can be considered as boundary area, see in more details [2]. Analyzing these classes of simplexes, we have shown that clusters in TBA solutions

**Fig. 1:** Distributions of Voronoi cell volumes for systems of random spheres at low packing fractions. All distributions are normalized to the mean volume (vertical line at 1). Dashed line corresponds to a system of random points.

**Fig. 2:** Variance of normed Voronoi cell volume distributions as a function of packing fraction: for random hard spheres (black line), and for TMAO (red triangles), TBA (blue squares) and urea (green diamonds) dissolved in water.
Fig. 3: The mean size of the clusters in the system of random spheres (black line), and in the aqueous solutions of TMAO (red), and TBA (blue) Cutoff distances for including particles into cluster were 0.58 nm (a).

are more compact than of urea, where they are more linear or branched. In the both cases they differ from the TMAO solutions, where the solute is randomly distributed.

Such analysis gives new information to chemists for understanding physical-chemical properties of solutions, and invites statisticians to apply there method to a broad field of nontrivial spatial systems: chemical solutions.

Financial support from grant RFFI (No. 15-03-03329) is gratefully acknowledged.


Densest Local Structures and Packing Properties of Uniaxial Ellipsoids

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The relationship between local structure and macroscopic properties is a current research focus in granular matter. We study the distribution of local packing fractions (defined via Set Voronoi cell volumina [2]) which is a sensitive observable for characterizing these systems. Previously, mainly packings of spherical particles were considered. Here, we focus on packings of uniaxial ellipsoids as an instance of aspherical particles [1]. In particular, we generalize the famous “kissing problem”, see Fig. 1, and report numerical results on the densest local structures of ellipsoids with aspect ratio $\alpha$ between 0.7 (oblate) and 1.4 (prolate), see Fig. 2.

Like in the spherical case, these packings locally exceed the density of known ellipsoid crystals [3]. Analogous to results for lattice packings [4], we find that el-
Fig. 2: Evolution of the densest structural motifs with aspect ratio $\alpha$. In addition to the number of particles in each ring (color-coded boxes), we give the point group for each structure. The oblate structures marked $N = 15$ and $N = 14$ have no nontrivial symmetries. ico refers to the icosahedral cluster which can be viewed either as 1-5-5-1 or 3-3-3-3 structure, see Fig. 51 top row. However the 1-5-5-1 structures (gray) never exceed the 3-3-3-3 in packing fraction.

Ellipsoids pack denser than spheres, and with more neighbors. In dense disordered packings of ellipsoids, distorted variations of some of our densest packing motifs can be identified. Our results permit us to generalize and test the $k$-Gamma model for local packing fraction distributions, previously only applicable to spherical particles [5].

Shape universality classes in the random sequential adsorption of non-spherical particles

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Random sequential adsorption (RSA) of particles of a particular shape is used in a large variety of contexts to model particle aggregation and jamming. A key feature of these models is the observed algebraic time dependence of the asymptotic jamming coverage $\phi_\infty - \phi(t) \sim t^{-\nu}$ as $t \to \infty$. However, the exact value of the exponent $\nu$ is not known apart from the simplest case of the RSA of monodisperse spheres adsorbed on a line (Renyi’s seminal ‘car parking problem’), where $\nu = 1$ can be derived analytically. Empirical simulation studies have conjectured on a case-by-case basis that for general non-spherical particles $\nu = 1/(d + \tilde{d})$, where $d$ denotes the dimension of the domain and $\tilde{d}$ the number of orientational degrees.

Plot of simulation results for the asymptotic scaling for a set of shapes with aspect ratio 1.5. Shown is the function $\log(\phi(2t) - \phi(t))$, which exhibits the same scaling as $\log(\phi(\infty) - \phi(t))$ when plotted against $\log(t)$. Ellipses show the predicted scaling $\nu = 2/3$. For rectangles, discorectangles, and spheroids the empirical exponent falls in the range $1/2 \leq \nu \leq 2/3$ indicating an intermediate time regime as explained by the theory. Data for $d = 1$ ($\tilde{d} = 2$) shapes are averaged over 500 (200) samples.

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of freedom of a particle. Here, we solve this long standing problem analytically for the $d = 1$ case — the ‘Paris car parking problem’. We prove that the scaling exponent depends on particle shape, contrary to the original conjecture, and falls into two universality classes [1]: (i) $\nu = 1/(1 + \bar{d}/2)$ for shapes with a smooth contact distance, e.g., ellipsoids; (ii) $\nu = 1/(1 + \bar{d})$ for shapes with a singular contact distance, e.g., spherocylinders and polyhedra. The exact solution explains in particular why many empirically observed scalings fall in between these two limits.

ACKNOWLEDGMENTS

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**Friday 15 September 2017**

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Processes of spatial points with multiple noisy appearances

Eva B. Vedel Jensen

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The analysis of patterns of points, subject to noise, is a well-studied topic in spatial statistics. Motivated by applications in PALM (PhotoActivated Localization Microscopy), we consider the more general situation where each parent point gives rise to multiple noisy appearances. To make inference for such point processes, in particular for the underlying parent process, we study the relationship between first- and second-order characteristics of the observed process and the parent process. We furthermore introduce parametric models for such point patterns, obtained in two steps by first generating a realization of a parametric model for the parent points and then associating to the parent points i.i.d. point processes. The focus is on double Cox processes for which both the parent process and the observed process are Cox processes. Parameter estimation for this type of model, based on moment methods and Bayesian inference, is discussed in the case where the underlying parent process is latent. The proposed methods are illustrated with an analysis of a data set from PALM microscopy.

Joint work with Ina Trolle Andersen, Eva C. Arnspang, Ute Hahn and Lene Niemann Nejsum.

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Let $V$ be a finite-dimensional euclidean vector space. We define a space $\text{Area}^{sm}$ of smooth area measures. An element $\Phi$ in this space is a translation invariant valuation on the space of compact convex bodies with values in the space of signed measures on the unit sphere (and satisfying some technical conditions). For instance, the classical area measures $S_k(K, \cdot), 0 \leq k \leq n-1$ are smooth area measures.

More generally, if $G$ is a subgroup of the euclidean group which acts transitively on the unit sphere, then the space $\text{Area}^G$ of smooth $G$-invariant area measures is finite-dimensional. If $\Psi_1, \ldots, \Psi_m$ is a basis of $\text{Area}^G$, then there are local additive kinematic formulas

$$\int_G \Psi_i(K + gL, \kappa \cap g\lambda) dg = \sum_{k,l} c^i_{k,l} \Psi_k(K, \kappa) \Psi_l(L, \lambda),$$

where $\kappa, \lambda$ are Borel subsets of the unit sphere. We define the operator $A_G : \text{Area}^G \to \text{Area}^G \otimes \text{Area}^G, \Psi_i \mapsto \sum_{k,l} c^i_{k,l} \Psi_k \otimes \Psi_l$.

To compute this operator explicitly, we use tensor valuations, i.e. continuous translation invariant valuations with values in the space $\text{Sym}^r V$ of symmetric tensors. The corresponding space is denoted by $\text{Val} \otimes \text{Sym}^r V$. By joint work with D. Hug, they satisfy a version of the additive kinematic formula.

The additive kinematic formulas for area measures and for tensor valuations are linked by the moment map. This is the map

$$M^r : \text{Area} \to \text{Val} \otimes \text{Sym}^r V, \quad \Phi \mapsto \left[ K \mapsto \int_{S^{n-1}} \bar{y}^r d(\Phi(K))(y) \right].$$

However, we must ensure that we do not lose information using the moment map. This follows from the first main theorem.

**Theorem 1:** The moment maps $M^r : \text{Area}_k \to \text{Val}_k \otimes \text{Sym}^r V, r \geq 2, 1 \leq k \leq n-1$ are injective.

The proof uses the Rumin differential operator and computations with differential forms.

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As an application, we introduce a certain space $\text{Area}^{*,sm}$ of smooth dual area measures. With $q_G$ being transposed to the inclusion $\text{Area}^G \to \text{Area}^{sm}$ and $A^*_G$ being transposed to $A_G$, we prove the following theorem, which is related to the convolution of valuations as defined in [2].

**Theorem 2:** There exists a natural convolution product on the space $\text{Area}^{*,sm}$ of smooth dual area measures such that for each transitive group $G$ the following diagram commutes

\[
\begin{array}{ccc}
\text{Area}^{*,sm} \otimes \text{Area}^{*,sm} & \xrightarrow{\ast} & \text{Area}^{*,sm} \\
\downarrow q_G \otimes q_G & & \downarrow q_G \\
\text{Area}^G \otimes \text{Area}^G & \xrightarrow{A^*_G} & \text{Area}^G
\end{array}
\]

Finally, we apply this theorem to write down the local additive kinematic formulas in the hermitian case $G = U(n)$ in a very explicit form. This generalizes recent papers of Wannerer [5, 6] and Solanes [4].

Self-assembly and growth of soft quasicrystals

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Quasicrystals possess long-ranged order but lack periodicity. They have attracted large attention during the last three decades due to their sophisticated material properties such as low surface friction and high rigidity. Meanwhile, soft quasicrystals built with colloids or polymer micelles constituents are in the focus of recent studies.

Interestingly, most quasicrystals observed in nature possess 5-, 8-, 10-, or 12-fold

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rotational symmetry. By studying colloidal quasicrystals we obtain new insights into the question why mainly these rotational symmetries occur in nature [1].

Another question that we explore is how the pair interaction of colloids has to be tailored in order to achieve the self-assembly of a quasicrystal with a desired symmetry. We explore the phase behavior of a colloid-polymer system where the colloids effectively attract each other at small distances due to depletion interactions but are repulsive for larger distances. Such a system leads to the self-assembly of a large variety of complex structures. For example, by tuning the density between a triangular phase and a square phase, we find a quasicrystal with 12-fold rotational symmetry (see Fig. 1(a)).

Furthermore, we study the consequences of the additional degrees of freedom that exist in quasicrystals and are called phasons. The phasons correspond to complex rearrangements of the colloids (see Fig. 1(b,c) or [2]). They do not cost free energy in the long-wavelength limit and therefore are hydrodynamic modes that are similar to phonons. As a consequence, phasons affect a lot of properties of the quasicrystal.

By employing a phase field crystal model we calculate how a quasicrystal grows from a seed and detect two different growth modes [3]. Close to the triple point a perfect, defect-free quasicrystal can grow (Fig. 2(a)). However, far away from the triple point the growth is dominated by phasonic flips which are incorporated as local defects into the grown structure (Fig. 2(b)). The later structure corresponds to a dislocation-free random-tiling-like quasicrystal.

Finally, we explore how the growth started by multiple seeds is affected by the possibility of stress relaxation via the phasonic degrees of freedom.
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Geometric functionals of fractal percolation

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Fractal percolation is a family of random sets suggested by Mandelbrot [1, 2] in the seventies as a model for turbulent dissipation in the oceans and the high atmosphere. It is one of the simplest scale invariant models and defined in terms of repeated subdivision of a square (or cube) \( Q \) in \( \mathbb{R}^d \). Given \( M \geq 2 \) and \( p \in [0, 1] \), \( Q \) is partitioned into \( M^d \) equal subsquares each of which survives the first step independently with probability \( p \) and is discarded otherwise. Then at each subsequent step, the procedure of division and erosion is repeated for each of the surviving subsquares of the previous step. The limit set of this procedure is called fractal percolation (or canonical curdling), cf. Fig. 1 for some realizations. Chayes, Chayes and Durrett [3] established that these models undergo a very sharp phase transition from a totally disconnected to a percolating regime, when (for fixed \( M \)) the continuous parameter \( p \) passes some threshold. The exact values of these percolation thresholds remain unknown until today, and the known rigorous upper and lower bounds are still rather far from each other, cf. e.g. [3, 4].

In the recent physics literature, see e.g. [5, 6], the idea is explored that the sharp topological transition at the threshold in percolation models should be visible in geometric functionals such as the (expected) Euler characteristic of these sets. Indeed, simulations suggest a close relation between percolation thresholds and the zeros of the Euler characteristic (as a function of the model parameter)

![Realizations of fractal percolation for different values of the parameter p](image)

*Fig. 1: Realizations of fractal percolation for \( M = 2 \) and different values of the parameter \( p \) (left: \( p = 0.8 \), middle: \( p = 0.9 \), right: \( p = 0.97 \)).

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in many percolation models. Motivated by the desire to find better bounds on percolation thresholds for fractal percolation, we study the expectations of some geometric functionals of these sets (and their rescaled limits). These functionals are closely related to fractal curvatures. We obtain explicit formulas for some of these expectations including some rescaled Euler characteristic and compare them to the known bounds for percolation thresholds.

Poster Contributions

Posters are displayed for the duration of the conference. There is a dedicated Poster Session on Tuesday evening from 8pm to 10pm.

Please take note of the Best Poster Award. Your conference booklet contains a ballot slip for your choice of the three best posters. Please hand in your completed ballot slip by the coffee break on Wednesday morning.
On the multiplicity of arrangements of equal zones on the sphere

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In d-dimensional Euclidean space $\mathbb{R}^d$ a plank of width $w$ is the closed region between two parallel hyperplanes that are of distance $w$ from each other. The famous plank problem of Tarski (1932) asks for the minimum total width of a collection of planks that can cover a given convex body $K$ (a compact convex set with non-empty interior). Here we investigate a problem on the $d$-dimensional unit sphere $S^{d-1}$ that can be considered as a relative of Tarski’s plank problem.

A spherical zone of (spherical) width $\omega$ is the intersection of $S^{d-1}$ with a Euclidean plank of width $2 \sin(\omega/2)$ that is symmetric to the origin. L. Fejes Tóth posed the following problem in 1973 (along with other questions) about spherical zones. What is the minimum width of $n$ congruent spherical zones that can cover $S^2$? He conjectured that the minimum width is equal to $\pi$, and that in the optimal configuration the central great circles of the zones are all incident with an antipodal pair of points and they are distributed evenly. This conjecture of L. Fejes Tóth was recently proved by Jiang and Polyanskii (2017) in a more general setting.

We investigate the multiplicity of arrangements (not only coverings) of $n$ congruent zones on $S^{d-1}$. Given an arrangement of zones on the sphere, its multiplicity is the largest positive integer $m$ such that there exists a point of the sphere that is contained in $m$ zones. Our goal is to minimize the multiplicity for given $d$ and $n$ as a function of the width of the zones.

In particular, we prove that, for sufficiently large $n$, it is possible to arrange $n$ equal zones of suitable width on $S^{d-1}$ such that no point belongs to more than a constant number of zones, where the constant depends only on the dimension and the width of the zones. We also prove that it is possible to cover $S^{d-1}$ by $n$ equal zones with multiplicity at most $A_d \ln n$, where $A_d$ is a constant depending only on $d$. These statements extend the corresponding earlier results of Frankl, Nagy and Naszódi (2016). We also investigate coverings of $S^{d-1}$ with equal zones with the property that each point of the sphere belongs to the interior of at most $d-1$ zones.

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Let $E \subset \mathbb{R}^d$ be some non-degenerate ellipsoid centered at the origin. For a fixed $k \in \{1, \ldots, d\}$ consider i.i.d. random vectors $X_1, \ldots, X_{k+1}$ uniformly distributed in $E$ and denote by $S_{k,d}(E) := \text{Conv}(X_1, \ldots, X_{k+1})$ their convex hull. We show that the expected volume of $S_{k,d}(E)$ coincides (up to an explicit constant) with the $k$-th intrinsic volume of $E$:

$$E \text{Vol}_k(S_{k,d}(E)) = c_{k,d}V_k(E).$$

We also generalize this relation to higher moments: for $p \geq 0$,

$$E \text{Vol}_k^p(S_{k,d}(E)) = c_{k,d,p} \int_{G_{d,k}} \text{Vol}_k^p(\text{Proj}_L[E]) \nu_{d,k}(dL),$$

where $G_{d,k}$ is the Grassmannian of all $k$-dimensional linear subspaces in $\mathbb{R}^d$ endowed with the standard uniform measure $\nu_{d,k}$. For $k = 1$, this result was obtained in [4]. The quantities in the right-hand side were introduced and studied in [2] and [3]. Using their results it is possible to rewrite (1) as

$$E \text{Vol}_k^p(S_{k,d}(E)) = c'_{k,d,p} E \left( \det AA^\top \right)^{p/2},$$

where $A$ is a random matrix whose rows are i.i.d. centered Gaussian vectors with dispersion ellipsoids coinciding with $E$. If $E$ coincides with the unit ball, then (2) reduces to the well-known result of Miles (see [1, Theorem 8.2.3]).

We also connect our results with random subsections of ellipsoids:

$$\int_{A_{d,k}} \text{Vol}_k^p(E \cap M) \mu_{d,k}(dM) = c''_{k,d,p} \text{Vol}_d(E)^{k+1} E \left( \det AA^\top \right)^{(p-d-1)/2},$$

where $A_{d,k}$ is the affine Grassmannian of all $k$-dimensional affine subspaces in $\mathbb{R}^d$ endowed with the standard motion invariant measure $\mu_{d,k}$.

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Phason modes in two-dimensional colloidal quasicrystals

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Colloidal particles which interact by a short-ranged double-well pair potential are known to assemble in quasiperiodic patterns [1]. We investigate an (approximant to the) ideal decagonal quasicrystalline order in two dimensions, and its deterioration under the effect of finite temperatures in the solid state, by Brownian Dynamics and Monte Carlo simulations.

In addition to continuous distortions of physical coordinates (phonons), quasicrystals possess hydrodynamic degrees of freedom which are called *phasons*. In particulate quasicrystals, phasonic distortions manifest in *flips*, characteristic jumps preserving the local quasiperiodic arrangement. The minor local energy penalty permits the exploration of phasonic degrees of freedom, driving competition of configurational entropy with global phasonic order. It is not known to what degree thermally excited phasonic flips occur in a collective manner, *i. e.* represent cooperative behaviour that favours the conservation of geometrical rules of the quasicrystal beyond the reach of the pair potential.

We anneal decagonal colloidal quasicrystals with Brownian Dynamics simulations. Thermal excitation of phononic displacements and phasonic flips are identified by embedding the quasicrystals into higher-dimensional space. Finite phononic strains are found to be pinned by flips even after cooling down to zero temperature, pointing to an interplay between phasons and phonons [2].

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Employing a multi-mode harmonic ansatz for an underlying phasonic excitation field, we relate the flip patterns to coherent plane-wave modes. For artificial phason waves, we observe decay into thermal equilibrium. In thermalised systems, however, the limited accuracy of a restricted set of plane waves questions the adequacy of a harmonic model for thermal phasonic excitations.

Since long-wavelength phasonic modes are inefficient to thermalise, we equilibrate phasonic excitations via Monte Carlo simulations with explicit flip vector moves. Phasonic flips do cause extensive phasonic centre-of-mass diffusion, but cannot destabilise the global order of the quasicrystal.

At low temperatures, the geometrical restriction on flip activity leads to transient flip dynamics within directed channels, corresponding to sub-diffusive transport properties. As a further consequence, the temperature dependence of the equilibrium profile of phasonic coordinates vanishes.

Recent progress in global-balance Monte Carlo algorithms has allowed to confirm the essentials of the Halperin-Nelson-Young theory (KTHNY) for the 2D Melting problem with short-range interactions [1]. A key challenge in these simulations are large correlation lengths which could be overcome by a new class of global-balance, rejection-free Monte Carlo algorithms, following the ‘Event-chain’ paradigm [2].

This poster shows that Event-chain Monte Carlo algorithms can be extended to include long-range forces (including periodic images) rigorously, without truncation the interaction potential. The new algorithm improves on the scaling of Ewald summation [3], which enables significantly increased particle numbers in systems with charged or polar interactions.

We can quantify the speed-up with respect to conventional reversible Monte Carlo methods and identify three distinct dynamical universality classes.

As an initial physics application, we present results on the 2D melting problem in the long-range limit, and complete the phase diagram of inverted power-law potentials, relevant for charged colloids, plasma crystals, and other systems. It is

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also possible to explicitly demonstrate the absence of long-ranged order predicted by the Mermin-Wagner theorem.

Stochastic reconstruction for inhomogeneous point processes

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In stochastic geometry visualisation and simulation studies remain fundamental part of point pattern analysis. Visual inspection of our data provides a useful preliminary step towards understanding its properties. Next step consists of finding a suitable model for observed pattern. Then with estimated parameters the model is fitted to the data and tested. Once the model is chosen, we can start with simulations that can be used for a number of different aims, e.g. exploring the sampling variation of estimated summary characteristics. However, this approach demands a lot of probabilistic and statistical knowledge.

Stochastic reconstruction lets us generate point patterns with prescribed summary characteristic that are free of explicit model conditions. This algorithmic procedure was first described in [2] in the context of reconstructing heterogeneous materials. Nowadays the method of stochastic reconstruction is of particular interest in biology and ecology. It can be used when monitoring forest ecosystems, see [3]. Of course, many other applications of this technique can be found in recent scientific papers, for example the quasi-plus sampling method by Tseschel and Chiu in [4] or the model-free isotropy test by Wong and Chiu, see [5]. Up to this day only stationary patterns were reconstructed. In this case we have a large choice of different summary characteristics. In the inhomogeneous case the situation becomes more complex because we are not able to intuitively define their inhomogeneous counterparts. The inhomogeneous version is known for K and J-function. Unfortunately, it is not clear whether the known counterparts of J and K-function contain enough information for successful reconstruction of observed point pattern.

Our aim is to discuss the possibility of the extension of the stochastic reconstruction algorithm described in [1] for second-order intensity-rewighted stationary point processes using the direct method of minimizing the energy functional (see [1]) and alternatively the Metropolis-Hastings algorithm.

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Low-density foams can be perceived as a subdivision of space into random (slightly curved) polyhedral cells whose edges form an interconnected network. The local structure of the cells obeys Plateau’s laws, that is, only three faces may intersect at a dihedral angle of exactly 120°. To obtain a simplified model, we neglect the curvature of the cells. A foam then resembles a subdivision of space in convex polyhedra whose local topology is induced by Plateau’s laws. In stochastic geometry such a subdivision of space is known as normal tessellation. It can be shown that any normal tessellation of the Euclidean space $\mathbb{R}^n$ with $n \geq 3$ is a Laguerre tessellation [2, 9].

A Laguerre tessellation is an additively weighted generalisation of the well known Voronoi tessellation. The latter is generated by a locally finite set $\varphi = \{x_1, x_2, \ldots\} \subset \mathbb{R}^n$ by assigning to each point $x \in \varphi$ the polyhedron $C(x, \varphi)$ composed of those points in space that have $x$ as nearest neighbour in $\varphi$. To generalise this notion, we attach weights $w \in \mathbb{R}$ to the points of $\varphi$. The corresponding Laguerre polyhedron of each pair $[x, w] \in \varphi$ is then defined by

$$C([x, w], \varphi) = \{ y \in \mathbb{R}^n : ||y - x||^2 - w^2 \leq ||y - x'||^2 - w'^2 \text{ for all } [x', w'] \in \varphi \}.$$ (1)

If all weights are equal, we obtain the Voronoi polyhedron as special case.

Random Laguerre tessellations appear to be a natural model for low-density foams as both share the same topology and a similar random cellular morphology. Indeed, we may consider a foam as a ‘relaxed’ Laguerre tessellation such that it fulfils Plateau’s laws [1, 7]. Physically this means to reduce free energy by minimising the surface area of the cells of a Laguerre tessellation by eliminating tiny edges and faces as illustrated in Fig. 41. A software to solve this and other problems involving surface minimisation is Brakke’s *Surface Evolver* [3]. Unfortunately, generating simulated foams is computationally expensive as the whole foam evolution process has to be modelled in software—including lots of topological transitions that result in cell neighbour switching [4–6].

In practice it would be desirable to avoid such complex simulations. Thus, we study how good an approximation of a foam by a Laguerre tessellation can be. For this purpose we compute a set of weighted points $y_i = \{[x_i, w_i]\}_{i=1}^m$.
Effect of the relaxation process transforming a Laguerre polyhedron (left) into a foam cell (middle). The Laguerre approximation of the foam is shown on the right. Note the absence of the small red faces of the Laguerre polyhedron in the foam cell and its approximation. The bottom row depicts the edge length distribution of the corresponding tessellation.

whose Laguerre tessellation best fits the cell system of a foam by minimising the discrepancy $f$ between the individual cells of the foam $P_i$ and the Laguerre tessellation $C_i$ [10], i.e.

$$f(y_1, \ldots, y_m) = \sum_{i \neq j} \int_{C_i \cap P_j} 1 \text{d}^n z \rightarrow \min y_i.$$ (2)

We call the resulting tessellation Laguerre approximation.

This approach provides an exact reconstruction of a tessellation if it is known to be a Laguerre tessellation. However, as foam cells have curved surfaces, an exact representation by Laguerre polyhedra is impossible. Nevertheless, the results in [10] show that the geometric cell properties (see [10, Tab. 1] for volume, surface area and mean breadth), edge length distribution (see Fig. 41) as well as the local topology (see [10, Fig. 8]) are reproduced quite well by Laguerre approximations. In [12] we considered the mechanical response of Laguerre approximations compared to open cell foams. For our study we generated 54 foams with 1728 and 2197 cells whose polydispersity ranged from 0 to 0.6. The polydispersity $p$ is defined as the quotient of the standard deviation $\sigma_r$ and the mean $\bar{r}$ of the equivalent sphere radius $r$, i.e. $p = \sigma_r / \bar{r}$, where $r$ is determined from the volume $V = \frac{4}{3} \pi r^3$ for each cell of the foam. To study the influence of the approximation on elasticity, we determined the difference $E_{\text{diff}}$ of Young’s modulus for each foam and its corresponding Laguerre approximation. In both cases, Young’s modulus was computed by connecting the vertices with straight beam elements of uniform Plateau-border cross section. $E_{\text{diff}}$ is depicted in Fig. 42.

For low-polydisperse foams with $0 \leq p < 0.25$, we found the elastic behaviour of the Laguerre approximations in very good agreement with the one of the foams. The approximations were at most 5% stiffer than the foams. For mid-polydisperse foams with $0.25 \leq p < 0.5$, the error in Young’s modulus increases to 12% following a cubic function $g(p) = 60.65p^3 + 4.26$ in $p$. This result
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Increase in Young’s modulus $E_{\text{diff}}$ when approximating foams by Laguerre tessellations w. r. t. polydispersity $p$.

indicates that with increasing polydispersity the influence of cell curvature on the elastic behaviour becomes predominant as the vertices of a face may not lie in a common plain. In practice, however, foams barely exceed a polydispersity of 0.3.

In summary our studies show that it is possible to realistically model (low-density) foams by Laguerre tessellations. Unfortunately, there is no parametric approach to fit such realistic models yet. Therefore sphere packing based Laguerre tessellations are commonly applied in foam modelling [8], albeit resulting in a non-realistic edge length distribution as shown in Fig. 41. To develop a more realistic foam model, it is necessary to understand the structure of the generating sets of Laguerre approximations. The approach presented in [10] provides the means to study this structure.

A first analysis of the generating sets of foams in [11] revealed a very regular structure with a high degree of correlation between the individual weighted points. Achieving a high regularity without suffering crystallisation effects is generally complicated and makes the modelling of foams a challenging task. An elegant modelling ansatz is Ords process [14, Discussion], where the interaction of points is described by the associated Voronoi tessellation. The generalisation of this model to Laguerre tessellations is a promising approach to generate tessellations similar to foams without simulating the foams themselves. This topic is subject of ongoing research.

Posters

Particles on surfaces are found both in nature and in the lab in the form of e.g. proteins in membranes or colloids adsorbed to a substrate or an interface. Curved surfaces are particularly interesting, because (i) typically these surfaces simply are not flat in nature and (ii) new physics may arise especially in the case of finite surfaces such as the surface of a sphere. Disks/spheres on spherical surfaces have been considered in experiments, (numerical) continuum theory and computer simulations [1–6]. More recently, also non-spherical particles on spherical surfaces were considered [7, 8]. Nevertheless, most of these works deal with spheres/disks on a sphere at infinite pressure (close packing); in contrast, we will consider the more interesting case of finite pressure, other particles than disks/spheres and other background spaces [9].

Here, I will present a classical density functional theory for two-dimensional hard particles on a two-dimensional curved surface. The theory is similar to the highly successful fundamental measure theory for three dimensional systems. The derivation is particularly elegant in two dimensions; we do not require any adjustable constants. We obtain analytical results for the homogeneous fluid of arbitrary convex particles on a surface with homogeneous curvature; the only two surfaces of this kind are the sphere and the hyperbolic plane. The latter surface can be approximately embedded in three dimensional space as a triply periodic minimal surface such as the gyroid, which can be formed by lipid membranes. The theory also applies to inhomogeneous systems, but the numerical implementation is

The classical density functional theory is derived for convex particles on a curved surface. For the sphere and the hyperbolic plane, the theory yields analytical results (also for non-circular particles).
nontrivial in that case and has not yet been performed. We compare to available theoretical or computer simulation results [1, 2, 9]. Previous theoretical approaches are either numerical or not as general as our theory. The analytic equation of state allows one to calculate, for example, the number of small spheres absorbed to a larger sphere in chemical equilibrium with a homogeneous fluid of small spheres with a known density (as is generally the case in experiments).

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A Central Limit Theorem for Lipschitz-Killing Curvatures of Gaussian Excursions

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Let $X = \{X(t) \mid t \in \mathbb{R}^d\}$ be a real, stationary, isotropic Gaussian random field. The excursion set of $X$ for the level $u \in \mathbb{R}$ is the random set

$$X^{-1}([u, \infty)) = \{t \in \mathbb{R}^d \mid X(t) \geq u\}.$$

The aim is to show a central limit theorem for the standardized Lipschitz-Killing curvature, also known as intrinsic volume, $L_m$, $m = 0, \ldots, d - 1$, of the intersection of the excursion set with an observation window as the window grows to the $d$-dimensional Euclidean space, that is

$$\frac{L_m\left(B_N^d \cap X^{-1}([u, \infty))\right) - \mathbb{E}\left[L_m\left(B_N^d \cap X^{-1}([u, \infty))\right)\right]}{\mathcal{H}^d(B_N^d)^{\frac{1}{2}}} \xrightarrow{\mathcal{D}} \mathcal{N}(0, \sigma_m^2)$$

as $N \to \infty$, where $\mathcal{H}^d(B_N^d)$ denotes the volume of the centered ball of radius $N$. Moreover a lower bound for the asymptotic variance $\sigma_m^2$ is presented.

Cluster counting in the random connection model

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The random connection model can be obtained by a stationary Poisson process η on \( \mathbb{R}^d \) and a connection function \( \varphi: [0, \infty) \to [0, 1] \). Each pair of points \( (x, y) \in \eta^2 \), with \( x \neq y \), is connected via an edge with probability \( \varphi(|x - y|) \), independently of all other points and connections. Here, the connection probability depends only on the distance between the points. This yields the random connection model \( \Gamma_\varphi(\eta) \), an undirected random graph with vertex set \( \eta \); see [2]. A well-known example of this model is the Gilbert graph (or random geometric graph) with constant radii. To construct the Gilbert graph, each point is marked with a constant radius and two points are connected if the balls around them intersect. This model was introduced in [3] and extensively studied in [4].

In the following let \( G \) be a connected graph with \( k \in \mathbb{N} \) vertices, that occurs as component in \( \Gamma_\varphi(\eta) \) with positive probability. We can label the components of the random connection model, that are isomorphic to \( G \) by their lexicographic minima. Let \( \eta_{\varphi,G} \) be the resulting stationary point process. For a compact and convex observation window \( W \subset \mathbb{R}^d \) (with non-empty interior) we have

\[
\eta_{\varphi,G}(W) = \#\{ \text{components of } \Gamma_\varphi(\eta) \text{ isomorphic to } G \\
\text{with lexicographic minimum in } W \}
\]

\[
= \sum_{(x_1, \ldots, x_k) \in \eta^k} 1\{ x_1 \in W, x_1 < \cdots < x_k \} 1\{ \{x_1, \ldots, x_k\} \text{ is a component of } \Gamma_\varphi(\eta) \text{ isomorphic to } G \},
\]

where \( < \) denotes the lexicographic order on \( \mathbb{R}^d \). Fig. 1 illustrates this situation.

Our main result is a quantitative central limit theorem for this counting variable as the size of the observation window is increasing. We show that

\[
d_K\left( \frac{\eta_{\varphi,G}(W) - \mathbb{E}\eta_{\varphi,G}(W)}{\text{Var}(\eta_{\varphi,G}(W))^{1/2}}, N \right) \leq \frac{c}{\lambda_d(W)^{1/2}}, \tag{1}
\]

where \( N \) is a standard normal random variable, \( d_K \) denotes the Kolmogorov distance and \( c > 0 \) is a positive constant. Inequality (1) is valid for all connection

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Fig. 1: The figure shows a realisation of a random connection model and the observation window $W$, given by the blue square. If $G$ consists of two connected vertices, we have $\eta_{\varphi,G}(W) = 3$. In addition to the two copies of $G$ in the interior of the observation window, the copy at the right margin of $W$ is also counted, since its lexicographic minimum is in $W$.

functions $\varphi$ with $\int \varphi(|x|)^{1/3} dx < \infty$ and all observation windows $W$ with sufficiently large inradius. In the special case of a connection function with bounded support, a qualitative central limit theorem was proved in [5]. The proof of (1) is based on general results for functions of edge marked Poisson processes, especially on a second order Poincaré inequality, that extends some results in [6].

Integer quantum Hall transitions on random Voronoi-Delaunay lattices

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The integer quantum Hall (QH) transition is a fascinating phenomenon in the realm of Anderson transitions. It occurs in two-dimensional (2D) electronic systems with a perpendicular magnetic field. In the applied field, the electrons undergo cyclotron motion which leads to discrete kinetic energy levels, called the Landau levels. Disorder broadens these levels into Landau bands and localizes all electronic states except one state in the each band center that remains extended. When the Fermi energy is tuned through one of the extended states, the system undergoes a quantum phase transition between two localized phases. These QH transitions are characterized by the critical exponent \( \nu \) of the localization length. Computational studies of integer QH transitions are often based on the semi-classical Chalker-Coddington (CC) network model. In recent years, several investigations obtained \( \nu \approx 2.60 \), see e.g. [1]. Experimental values are significantly smaller, \( \nu \approx 2.38 \) [2], suggesting that the integer QH transitions in the CC network and in real systems belong to different universality classes. This could be caused by the electron-electron interactions that are unavoidable in real systems but neglected in the CC network. However, Gruzberg et al. recently proposed that the disorder in the usual CC network does not fully reflect the disorder in a real systems and is thus not sufficient to correctly describe the QH transition [3]. They constructed a geometrically disordered version of the CC model and obtained \( \nu \approx 2.37 \), which implies a different universality class than the usual CC model.

To shed light onto this question, we consider a microscopic model of noninteracting electrons rather than the CC network. Specifically, we consider a 2D tight-binding model with perpendicular magnetic field. It is defined on a random Voronoi-Delaunay (VD) lattice in which bonds between randomly positioned sites are obtained by the Delaunay triangulation (see Fig. 1, left panel). The resulting topologically disordered lattice features strong anticorrelations between the coordination numbers of neighboring sites. This modifies the Harris and Imry-Ma criteria and leads to qualitative changes of the scaling behavior at magnetic phase transitions [4]. The wave functions \( \Psi \) of the electrons fulfill the Schrodinger

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The Hamiltonian matrix in site basis,

\[ H = \sum_{\langle m,n \rangle} \exp (i\varphi_{mn}) |m\rangle \langle n| \],  \tag{1} \]
does not have diagonal disorder, and its only off-diagonal elements are given by the bonds of the random VD lattice. The associated bond strengths are values on the complex unit circle, where the spatial positions \((x_n, y_n)\) of the bond sites and the strength of the magnetic flux \(\Phi\) form the (Peierls) phase

\[ \varphi_{mn} = 2\pi \Phi \frac{x_n + x_m}{2} (y_m - y_n) \]. \tag{2} \]

The disorder which is necessary for the phase transitions stems from the connectivity of the random VD lattice and the from the random Peierls phases arising from Eq. (2). In the absence of a magnetic field, the random connectivity causes all states to be localized [5]. For nonzero magnetic field, in contrast, we find several QH transitions. We analyze the critical behavior of the lowest Landau level by means of a recursive Green function approach. Here, we calculate the smallest positive Lyapunov exponent \(\gamma\) describing the long-range behavior of wave function intensities along a quasi-one-dimensional VD lattice stripe. The observed critical behavior depends on the strength of the magnetic field. For an intermediate flux, all critical parameters agree well with the latest results of the (generic) CC model (see Fig. 1, right panel) [1].

Fig. 1: Left: 2D random VD lattice for 100 randomly positioned sites (dots) placed in a square (red lines) of linear size \(L_x = L_y = 10\). The Voronoi cells (polygons) of the original system are colored arbitrarily and periodically repeated cells are grayed. The lines show the Delaunay triangulation which connects lattice sites directly (solid black lines) and under use of periodic boundary conditions (dashed black lines). Right: Behavior of the dimensionless Lyapunov exponent \(\Gamma \equiv \gamma L\) in vicinity of the critical point for several \(L\). Each data point is based on 50 lattice stripes with dimensions \(L_x = 10^6\) and \(L_y = L\). Solid lines result from scaling analysis with \(\Gamma_c \approx 0.81\) and \(\nu \approx 2.60\).
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On a class of sets admitting curvature measures

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A class of closed subsets of $\mathbb{R}^d$ called WDC sets was introduced recently; these sets extend significantly the Federer’s sets with positive reach and still admit curvature measures fulfilling the basic integralgeometric formulas (see [1]). The WDC sets are defined analytically as weakly regular sublevel sets of DC functions (differences of convex functions) and it seems to be a difficult task to obtain any geometric description of these sets. Our aim is to present some partial results in this directions which were obtained in [2]. In dimension 2, a full characterization of WDC sets is available. We are also able to characterize WDC sets among Lipschitz manifolds (closed Lipschitz domains) as DC manifolds (closed DC domains, respectively). Also, a classification of points of a WDC sets as “regular” and “irregular” is suggested, and the set of irregular points is shown to be small in certain sense.

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Tomographic analysis of bidisperse packings of ellipsoids

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Microscopic (grain-scale) models for granular materials need to incorporate physical parameters that are pertinent at the particle scale such as friction coefficients, compressibility, etc. They also need to take into account distribution properties such as the variability in grain size and shape.

We aim to reduce the model complexity to essential ingredients by identifying the key mechanisms at play that govern the granular compaction. The key question we aim to address is the extension of the ‘zero-th order’ model—the spherical case—to model with first- and second-order complexity. Recent years have seen many studies addressing the effects of the particle shape while maintaining uniform particles. We focus on a different effect, namely the effect of the grain size distribution.

We report experimental results on granular compaction in a model system made of mono- and bidisperse ellipsoidal packings as well as sand packings with grain size polydispersity. The packings are subject to vertical tapping of varying duration (number of taps) and different tapping protocols. Their internal three-dimensional structure is obtained using x-ray computed tomography. Particles

Bidisperse spheres (left), bidisperse ellipsoids (middle) and polydisperse Ottawa sand (right).

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positions and orientations are reconstructed and the global packing densities are computed. The analysis of the vertical and horizontal local packing fraction profiles reveal a homogeneous densification in the ellipsoidal packings, however, sand packings exhibit radial density gradient, possibly linked to the onset of convection.

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Driven granular matter exhibits a rich variety of nonequilibrium phases [1, 2]. Recently, a critical transition to a state with quadratic order has been reported, with several critical exponents measurable [3]. We study this set-up by computer simulations, which consists of spherical particles between two horizontal plates. The particles are agitated by vibrating the plates in vertical direction. The energy injection is balanced by energy loss through inelastic collisions of the granular particles. Thus, the system reaches a steady state which exhibits phase behavior similar to equilibrium systems. The gap between the plates is about two particle diameters allowing the particles to form - besides fluid-like states - hexagonal and quadratic bilayers. We determine the relevant parameters for formation of ordered states, present a numerical phase diagram for this system, and study phase coexistence and criticality.

Upper bound on the Edwards entropy in frictional hard-sphere packings

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We extend the Widom particle insertion method [1] to determine an upper bound \( s_{ub} \) on the Edwards entropy in frictional hard-sphere packings. \( s_{ub} \) corresponds to the logarithm of the number of mechanically stable configurations for a given volume fraction and boundary conditions. To accomplish this, we extend the method for estimating the particle insertion probability through the pore-size distribution in frictionless packings [2] to the case of frictional particles. We use computer-generated and experimentally obtained three-dimensional sphere packings with volume fractions \( \phi \) in the range 0.551 - 0.65. We find that \( s_{ub} \) has a maximum in the vicinity of the Random Loose Packing Limit \( \phi_{RLP} = 0.55 \) and decreases then monotonically with increasing \( \phi \) to reach a minimum at \( \phi = 0.65 \). Further on, \( s_{ub} \) does not distinguish between real mechanical stability and packings in close proximity to mechanical stable configurations. The probability to find a given number of contacts for a particle inserted in a large enough pore does not depend on \( \phi \), but it decreases strongly with the contact number. The results on this poster have been published in [3].


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Visualization of the Widom method using a one-dimensional hard-sphere system, and ignoring the condition of mechanical stability. Panel a) shows a specific two-particle system in a one-dimensional box extending from 0 to 1. Panel c) displays the available phase space $p_{\text{correct}}(2,V)$, for arbitrary positions of the two particles, as a green area. The specific system of panel a) is here indicated as a black dot. Panel b) visualizes in green the available space for the insertion of a third particle in this specific system without overlap with the first two particles. The green area in panel d) shows the available phase space of the generic three-particle system. Panel b) corresponds to an one-dimensional hyperslice of this phase space, it is indicated by the black vertical line. The Widom insertion method now corresponds to the statement that one can compute $p_{\text{correct}}(3,V)$ (the fraction of the green volume in 3D in panel d)) by multiplying $p_{\text{correct}}(2,V)$ (the fraction of green area in panel c)) with $p_{\text{insert}}$ (the fraction of the green length in panel b)). Image courtesy to Sibylle Nägle.
Tensorial Curvature Measures in Integral Geometry

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The intrinsic volumes $V_j : \mathcal{K}^n \to \mathbb{R}, j \in \{0, \ldots, n\}$, are defined as the coefficients of the monomials in the Steiner formula

$$\mathcal{H}^n(K + \varepsilon B^n) = \sum_{j=0}^{n} \kappa_{n-j} V_j(K) \varepsilon^{n-j},$$

for a convex body $K \in \mathcal{K}^n$ and $\varepsilon \geq 0$.

For a polytope $P \in \mathcal{P}^n$ the intrinsic volume $V_j$ is explicitly given as the sum of the volumes of the $j$-dimensional faces of $P$ multiplied with the exterior angle at each face, i.e.

$$V_j(P) = \frac{1}{\omega_{n-j}} \sum_{F \in \mathcal{F}_j(P)} \mathcal{H}^j(F) \mathcal{H}^{n-j-1}(N(P,F) \cap S^{n-1}).$$

In 1957, Hadwiger characterized the intrinsic volumes uniquely as the only continuous and motion invariant valuations on convex bodies.

The parallel set $K + \varepsilon B^n$ of a convex body $K \in \mathcal{K}^2$ is the union of all points in $\mathbb{R}^2$ of distance at most $\varepsilon > 0$. The parallel set $P + \varepsilon B^n$ of a convex polytope $P \in \mathcal{P}^2$ can be partitioned into the points contributing to $V_0(P)$ (green), $V_1(P)$ (red), and $V_2(P)$ (blue).

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The support measures $\Lambda_j : \mathcal{K}^n \times \mathcal{B}(\mathbb{R}^n \times \mathbb{S}^{n-1}) \to \mathbb{R}$, $j \in \{0, \ldots, n-1\}$, are defined by a local Steiner formula. They naturally localize the intrinsic volumes, i.e. $\Lambda_j(K, \mathbb{R}^n \times \mathbb{S}^{n-1}) = V_j(K)$, $K \in \mathcal{K}^n$. The curvature measures $\phi_j$, $j \in \{0, \ldots, n-1\}$, are the marginal measures on $\mathcal{B}(\mathbb{R}^n)$ of the support measures, and hence defined by $\phi_j(K, \cdot) := \Lambda_j(K, \cdot \times \mathbb{S}^{n-1})$. The curvature measures of a polytope $P \in \mathcal{P}^n$ are explicitly given by

$$\phi_j(P, \beta) = \frac{1}{\omega_{n-j}} \sum_{F \in \mathcal{F}_j(P)} \int_{F \cap \beta} \mathcal{H}^j(dx) \int_{N(P,F) \cap \mathbb{S}^{n-1}} \mathcal{H}^{n-j-1}(du).$$

For the given polytope $P \in \mathcal{P}^2$, and the Borel set $\beta \in \mathcal{B}(\mathbb{R}^2)$, only the faces $F_2$ and $F_5$ contribute to the 0th curvature measure $\phi_0$ of $P$ evaluated at $\beta$, as $F_i \cap \beta = \emptyset$ for $i \neq 2, 5$.

The tensorial curvature measures are a tensor-valued generalization of the curvature measures. For $r, s \in \mathbb{N}_0$, they are given by

$$\phi_j^{r,s,0}(K, \beta) := c_{n,j}^{r,s,0} \int_{\beta \times \mathbb{S}^{n-1}} x^r u^s \Lambda_j(K, d(x,u)),$$

where $c_{n,j}^{r,s,l} > 0$ is a normalizing constant and $x^r u^s \in \mathbb{T}^{r+s}$ is a symmetric tensor product, i.e. a symmetric multilinear mapping from $(\mathbb{R}^n)^{r+s}$ to $\mathbb{R}$.

On polytopes, we obtain the generalized tensorial curvature measures. They are given by

$$\phi_j^{r,s,l}(P, \beta) := \frac{c_{n,j}^{r,s,l}}{\omega_{n-j}} \sum_{F \in \mathcal{F}_j(P)} Q(F)^l \int_{F \cap \beta} x^r \mathcal{H}^j(dx) \int_{N(P,F) \cap \mathbb{S}^{n-1}} u^s \mathcal{H}^{n-j-1}(du),$$

for $r, s, l \in \mathbb{N}_0$, where $Q(F) \in \mathbb{T}^2$ denotes the metric tensor on $F$. For $l = 0$, the $\phi_j^{r,s,0}$ coincide with the tensorial curvature measures on convex bodies. For $l = 1$, there exists a continuous extension of the $\phi_j^{r,s,1}$ to arbitrary convex bodies. For $l > 1$, the $\phi_j^{r,s,l}$ can probably not be continuously extended to arbitrary convex bodies.
The generalized tensorial curvature measures are isometry covariant and locally defined tensor measure valued valuations, which are essentially linearly independent.

There are two fundamental intersection formulae in classical integral geometry, the so-called principal kinematic formula and the Crofton formula. These formulae express the integral mean of the intrinsic volume of the intersection of a given convex body with a second geometric object (another convex body in the kinematic case and an affine subspace in the Crofton case) which is uniformly moved by a proper rigid motion, in terms of linear combinations of the intrinsic volumes of the given geometric objects.

A fixed convex body $K \in \mathcal{K}^2$ is intersected with a convex body $K' \in \mathcal{K}^2$, moved by a rigid motion $g \in G_2$ in the principal kinematic formula, resp. with an affine subspace $E \in \mathcal{A}(2,1)$ in the Crofton formula.

**Principal Kinematic Formula:** For $K, K' \in \mathcal{K}^n$ and $0 \leq j \leq n$,

$$\int_{G_n} V_j(K \cap gK') \mu(dg) = \sum_{k=j}^{n} \alpha_{n,j,k} V_k(K)V_{n-k+j}(K'),$$

where $\alpha_{n,j,k} = \frac{\Gamma(\frac{k+1}{2})\Gamma(\frac{n-k+j+1}{2})}{\Gamma(\frac{j+1}{2})\Gamma(\frac{n+k+1}{2})}$. Here, $\mu$ denotes the invariant Haar measure on the motion group $G_n$.

**Crofton Formula:** For $K \in \mathcal{K}^n$ and $0 \leq j \leq k \leq n$,

$$\int_{A(n,k)} V_j(K \cap E) \mu_k(dE) = \alpha_{n,j,k} V_{n-k+j}(K),$$

where $\alpha_{n,j,k}$ is defined as in the principal kinematic formula. Here, $\mu_k$ denotes the invariant Haar measure on the affine Grassmannian $A(n, k)$ of $k$-dimensional affine subspaces of $\mathbb{R}^n$.

On the poster, we present two complete sets of kinematic and Crofton formulae, for the generalized tensorial curvature measures on polytopes. In these formulae, the kinematic (resp. Crofton) integral of the generalized tensorial curvature
measure is expressed as a linear combination of the generalized tensorial curvature measures of the given geometric objects. Due to the linear independence of the involved valuations, these representations can essentially not be simplified further.

These formulae yield further integral geometric formulae for the (generalized) tensorial curvature measures on arbitrary convex bodies and for the Minkowski tensors.

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Shape, Friction and Cohesion in Granular Packings

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Friction and adhesive forces are important parameters for the stability of granular packings. We analyze packings of wetting and non wetting spheres and triaxial ellipsoids with different aspect ratios. The structural properties of packings are analyzed with respect to friction and adhesive forces. Interparticle friction is changed by grinding the particles with different abrasives and by applying liquid and dry lubricants, which also changes adhesive forces. Adhesive forces are changed by adding water with a surfactant to the packing. Various packings with a range of friction coefficients and liquid contents are prepared at various packing fractions.

To obtain structural properties, the packings are recorded by X-ray tomography and particles as well as liquid clusters are detected. Structural characterization includes mean and local packing fractions, contact numbers as well as Voronoi cell anisotropy by Minkowski tensors.

We show that, although friction has an impact on the mechanical characteristics,

Voronoi tesselation of a sphere packing. The spheres have been replaced by colored ellipsoids, which resemble the local anisotropy, measured with the Minkowski Tensor $\beta_0^{2,0}$, of the respective cell.

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the analyzed local structural features remain unchanged.

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