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Institute for Applied and Numerical Mathematics

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Advanced Topics in Numerical Analysis I
Homework Assignment No. 1

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Problem 1 (no correction)

Consider the functions $f, g \in C^{n+1}[a,b]$ with

$$|f^{(n+1)}(x)| \le g^{(n+1)}(x) \text{ for all } x \in [a,b] \ .$$

Let p_f resp. p_g be the interpolation polynomial of f resp. g with respect to the nodes $a \le x_0 < x_1 < \cdots < x_n \le b$. Proof the following error estimate

$$|f(x) - p_f(x)| \le |g(x) - p_g(x)| \text{ for all } x \in [a,b] \ .$$

Instruction:
First proof the statement under the additional assumption $g^{(n+1)}(x) > 0$ for all $x \in [a,b]$ then proof the general case considering $g_\varepsilon(x) := g(x) + \varepsilon x^{n+1}$ for $\varepsilon \to 0$.

Problem 2 (no correction)

a) Let $p \in \mathcal{P}_n$ be the interpolation polynomial with respect to the equidistant nodes $x_j := x_0 + jh$, $h > 0$, and data y_j, $j = 0, 1, \ldots, n$. Verify the Gregory-Newton representation of p with forward differences $\Delta^k y_0$ resp. backward differences $\nabla^k y_n$

$$\text{(I)} \quad p(x) = \sum_{k=0}^{n} \binom{t}{k} \Delta^k y_0 \ , \qquad t = \frac{x - x_0}{h} \ ,$$

$$\text{(II)} \quad p(x) = \sum_{k=0}^{n} (-1)^k \binom{-t}{k} \nabla^k y_n \ , \qquad t = \frac{x - x_n}{h} \ .$$

b) Let $x_0, x_1, \ldots, x_n \in [a,b]$ be distinct and $\omega_{n+1}(x) := \prod_{\nu=0}^{n} (x - x_\nu)$. Proof for $f \in C[a,b]$ the representation

$$f[x_0, x_1, \ldots, x_n] = \sum_{j=0}^{n} \frac{f(x_j)}{\omega'_{n+1}(x_j)} = \sum_{j=0}^{n} \Big(\prod_{\substack{\nu=0 \\ \nu \ne j}} (x_j - x_\nu) \Big)^{-1} f(x_j) \ .$$

Problem 3 (no correction)

Given the nodes $x_0 = -2, x_1 = -1, x_2 = 0, x_3 = 1, x_4 = 2$ and the data $y_0 = 0, y_1 = 30, y_2 = 39, y_3 = 50, y_4 = 48$, we seek a rational function

$$R(x) = \frac{a_0 + a_1 x + a_2 x^2}{b_0 + b_1 x + b_2 x^2}$$

with $b_0 = 1$ such that the interpolation conditions

$$R(x_j) = y_j, \quad j = 0, \ldots, 4,$$

are satisfied.

a) Derive a interpolating rational function R and proof its uniqueness.

b) Draw the graph of the interpolating function R. Is R continuous?

c) If some data is changed, i.e. $y_2 = 40$ instead of $y_2 = 39$, how does that effect the problem?

Problem 4 (no correction)

Consider the function $f : [0,2] \to \mathbb{R}$, $f(x) := \cos(\pi x)$ and the nodes $x_0 = 0, x_1 = 1, x_2 = 2$.

a) Compute the Hermite interpolation polynomial p_5 of f, that interpolates f at the given nodes.

b) Give an estimate of the error $\max_{x \in [0,2]} |f(x) - p_5(x)|$
(see Problem 48, Numerical Mathematics I, SS 2006).

On **October 16, 14:00-15:30h** the problems will be discussed in the Architektur-Hörsaal No. 9 (building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also availaible for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .