Universität Karlsruhe (TH)
Research University • founded 1825

Institute for Applied and Numerical Mathematics

Prof. Dr. Rudolf Scherer, Dipl.-Math. Wolfgang Müller

Advanced Topics in Numerical Analysis I
Homework Assignment No. 2

(WS 2006/2007) October 26, 2006

Problem 5 (to be handed in)

Consider the quadrature formula

$$\int_0^1 f(x)\,dx = \frac{1}{2}\big(f(0)+f(1)\big) + \frac{1}{12}\big(f'(0)-f'(1)\big) + R(f)$$

with $f \in C^4[0,1]$.

a) Show that the quadrature formula is exact of order 3.

b) Compute the Peano kernel $K_3(t)$ in the representation of the error

$$R(f) = \int_0^1 K_3(t) f^{(4)}(t)\,dt \ .$$

c) Draw the graph of $K_3(t)$.

d) Proof the representation

$$R(f) = \frac{1}{720} f^{(4)}(\xi) \text{ with } \xi \in (0,1) \ .$$

Problem 6 (no correction)

Show that the Peano kernel G of a divided difference L_k with respect to the nodes $x_0 < x_1 < \dots < x_k$ has the following properties.

i) G is a spline of degree $k-1$ (with respect to $\Delta = \{x_0, x_1, \dots, x_k\}$);

ii) G is identical to zero in $(-\infty, x_0)$ and (x_k, ∞);

iii) G is strictly positive in (x_0, x_k);

iv) $\displaystyle\int_{x_0}^{x_k} G(t) = \frac{1}{k!}$.

Problem 7 (to be handed in)

Consider the nodes $x_0 = 0, x_1 = \frac{\pi}{3}, x_2 = \frac{2\pi}{3}$ and the function $f(x) := \cos^3 x$.

a) Compute the interpolation function $\psi_2 \in [\varphi_0, \varphi_1, \varphi_2] := [1, \cos x, \cos 2x]$ of f. First compute elementary functions $\eta_0, \eta_1, \eta_2 \in [\varphi_0, \varphi_1, \varphi_2]$ with

$$\eta_\nu(x_\mu) = \delta_{\nu\mu} \ , \qquad \nu = 0,1,2, \quad \mu = 0,\dots,\nu \ ,$$

and then the coefficients α_ν, $\nu = 0,1,2$, in the representation

$$\psi_2(x) = \sum_{\nu=0}^{2} \alpha_\nu \eta_\nu(x) \ .$$

b) The interpolation problem in a) is changed by adding the node $x_3 = \pi$ and the function $\varphi_3(x) = \cos 3x$. Compute the corresponding interpolation function ψ_3 analogously to a) and show that ψ_3 is equal to f.

Problem 8 (no correction)

Consider the trigonometric interpolation polynomial $T(x)$ of a function $f \in C_{2\pi}$ with respect to the nodes $x_j = \frac{j\pi}{m}$, $j = 0,1,\dots,2m-1$.

a) Show: If f is an even function, the interpolation polynomial $T(x)$ is even.

b) Give an interpretation of this result as a solution to the interpolation problem with respect to the Haar system $\{1, \cos x, \dots, \cos mx\}$ on $[0,\pi]$.

c) Using Tschebyscheff polynomials $T_k(x)$ derive a useful representation of the algebraic interpolation polynomial with respect to the nodes $t_j = \cos\frac{j\pi}{m}$ and the data y_j, $j = 0,1,\dots,m$ (see Problem No. 19, Numerical Mathematics I, SS 2006).

Please hand in your homework problems (No. **5** and **7**) due **Thursday**, **November 2, 2006**, **13:00h**. Put them in the slot marked „Numerische Mathematik I/II/III“ in the Math-Building (20.30), 2nd floor opposite room 112. Please print your name and registration number on your problems.

On **Thursday**, **November 2, 2006**, **14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .