Universität Karlsruhe (TH)
Research University • founded 1825
Institute for Applied and Numerical Mathematics
Prof. Dr. Rudolf Scherer, Dipl.-Math. Wolfgang Müller

Advanced Topics in Numerical Analysis I
Homework Assignment No. 7

(WS 2006/2007) November 30, 2006

Problem 25 (to be handed in)

Consider the quadrature formula of Romberg

$$T_{0,n} := h_n \sum_{j=0}^{2^n}{}'' f(a+jh_n) , \qquad \text{für } n=0,1,\dots ,$$
$$T_{m,n} := \frac{4^m T_{m-1,n+1} - T_{m-1,n}}{4^m-1} \qquad \text{für } m=1,2,\dots ,$$

with $h_n := (b-a)2^{-n}$. Show that the quadrature formula is stable. Prove by induction (over m) that the recursion formula

$$Q_{0,n} := 4T_{0,n+1} - 2T_{0,n} , \qquad \text{für } n=0,1,\dots ,$$
$$Q_{m,n} := \frac{1}{4^m-1}\left(2^{2m+1}T_{m-1,n+1} + 2T_{m-1,n} + 4^{m+1}Q_{m-1,n+1}\right) \qquad \text{für } m=1,2,\dots .$$

has the following properties:

a) $T_{m-1,n} + Q_{m-1,n} = (4^m-1)T_{m,n}$,

b) The quadrature weights of $T_{m,n}$ are positive. And the weights of $Q_{m,n}$ are non-negative.

Problem 26 (to be handed in)

Show:

a) The matrix A is normal ($A^HA = AA^H$) if and only if A is unitarily similar to a diagonal matrix. **Hint:** Theorem of Schur (Theorem 7.1)

b) If $U \in \mathbb{C}^{n\times n}$ is a unitary matrix then the range $G[A]$ of a matrix $A \in \mathbb{C}^{n\times n}$ is

$$G[U^HAU] = G[A].$$

c) Is A normal then $G[A]$ is the convex hull of the eigenvalues of A.

Problem 27 (no correction)

a) For which $x, y \in \mathbb{R}^n$ does a vector $w \in \mathbb{R}^n$, $w^Tw = 1$ exist such that the Householder transformation $H = I - 2ww^T$ maps the vector x onto y?

b) Compute for the matrices

$$A = uv^T, \qquad u,v \in \mathbb{R}^n \setminus \{0\},$$
$$B = I - 2ww^T, \qquad w \in \mathbb{R}^n, w^Tw = 1$$

eigenvalues, eigenvectors and (if necessary) principal vectors.

Problem 28 (no correction)

Consider the tridiagonal matrix

$$A = \begin{pmatrix} \delta_1 & \gamma_2 & & 0 \\ \beta_2 & \delta_2 & \ddots & \\ & \ddots & \ddots & \gamma_n \\ 0 & & \beta_n & \delta_n \end{pmatrix}.$$

Show:

a) λ is an eigenvalue of A if and only if $-\lambda$ is an eigenvalue of $B := A - 2\mathrm{diag}(\delta_1,\dots,\delta_n)$.

b) For the real matrix A, where $\beta_j = \gamma_j$, $j = 2,\dots,n$ and where

$$\delta_j = -\delta_{n+1-j}, \quad j = 1,\dots,n,$$
$$\gamma_j = \gamma_{n+2-j}, \quad j = 2,\dots,n$$

is $-\lambda$ an eigenvalue of A if λ is an eigenvalue of A.

c) For the matrix A, where $\delta_j = 0$, $j = 1,\dots,n$ and $\beta_j = \bar{\gamma}_j$, $j = 2,\dots,n$ the eigenvalues are symmetric to 0 and the determinant of A is

$$\det A = \begin{cases} (-1)^k|\gamma_2|^2|\gamma_4|^2\dots|\gamma_n|^2, & \text{if } n \text{ even}, \\ 0, & \text{if } n \text{ odd}. \end{cases}$$

Please hand in your homework problems (No. **25** and **26**) due **Thursday**, **December 7, 2006**, **13:00h**. Put them in the slot marked „Numerische Mathematik I/II/III“ in the Math-Building (20.30), 2nd floor opposite room 112. Please print your name and registration number on your problems.

On **Thursday**, **December 7, 2006**, **14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .