Universität Karlsruhe (TH)
Research University • founded 1825

Institute for Applied and Numerical Mathematics

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Advanced Topics in Numerical Analysis I
Homework Assignment No. 8

(WS 2006/2007) December 7, 2006

Problem 29 (to be handed in)

Let $\alpha, \gamma \geq 0$. Consider the tridiagonal Toeplitz matrix

$$T = \begin{pmatrix} \beta & \gamma & & & \\ \alpha & \beta & \gamma & & \\ & \ddots & \ddots & \ddots & \\ & & \alpha & \beta & \gamma \\ & & & \alpha & \beta \end{pmatrix} \in \mathbb{R}^{n\times n}.$$

Show:

a) T has the eigenvalues

$$\lambda_k = \beta + 2\sqrt{\alpha\gamma}\cos\frac{\pi k}{n+1}, \quad k = 1,\dots,n$$

and if $\alpha\gamma \neq 0$ the eigenvectors

$$v_k = (\delta\sin\frac{\pi k}{n+1}, \delta^2\sin\frac{2\pi k}{n+1}, \dots, \delta^n\sin\frac{n\pi k}{n+1})^T, \quad \delta := \sqrt{\frac{\alpha}{\gamma}}, \quad k = 1,\dots,n.$$

b) In the case $\alpha = \gamma = 1$ and $\beta = -2$ the condition number of T is given by

$$\text{cond}_2(T) = O(n^2)\ .$$

Problem 30 (to be handed in)

Consider the matrix

$$A = \begin{pmatrix} 6 & -2 & -2 & 1 \\ -2 & 6 & -1 & 1 \\ -2 & -1 & 10 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}.$$

a) Use the reduction method of Householder to transform the matrix A into a *tridiagonal matrix.*

b) Determine all eigenvalues of A.

Problem 31 (no correction)

a) Let $A \in \mathbb{R}^{n\times n}$ symmetric and $B \in \mathbb{R}^{n\times n}$ symmetric, positive definite. Show that the generalized eigenvalue problem

$$Ax = \lambda Bx$$

can be reduced to an equivalent eigenvalue problem

$$Cy = \lambda y$$

with a symmetric matrix $C \in \mathbb{R}^{n\times n}$.

b) Determine the corresponding matrix C to

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \\ -1 & 1 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix}.$$

Problem 32 (no correction)

a) Using the method of Hyman determine an approximation to an eigenvalue in the eigenvalue problem $Ax = \lambda x$ where

$$A = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 3 & -3 \\ 0 & -3 & 5 \end{pmatrix}.$$

Carry out one iteration step beginning with the starting value $t_0 = 0$.

b) Let $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times m}$.

Show that the nonzero eigenvalues of AB are exactly the nonzero eigenvalues of BA.

Please hand in your homework problems (No. **29** and **30**) due **Thursday**, **December 14, 2006**, **13:00h**. Put them in the slot marked „Numerische Mathematik I/II/III“ in the Math-Building (20.30), 2nd floor opposite room 112. Please print your name and registration number on your problems.

On **Thursday**, **December 14, 2006**, **14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .