Universität Karlsruhe (TH)
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Institute for Applied and Numerical Mathematics

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Advanced Topics in Numerical Analysis I
Homework Assignment No. 9

(WS 2006/2007) December 14, 2006

Problem 33 (no correction)

Consider the two Householder matrices $H_1 = I - 2uu^T, H_2 = I - 2vv^T \in \mathbb{R}^{n\times n}$ corresponding to the vectors $u, v \in \mathbb{R}^n$, $\|u\|_2 = \|v\|_2 = 1$.

a) Show that $H = H_1H_2$ is diagonalizable and that all eigenvalues of H are lying on the boundary of the unit circle.

b) Determine all eigenvalues and the corresponding eigenvectors of H.

Problem 34 (to be handed in)

Let $A \in \mathbb{C}^{n\times n}$ non-singular with the eigenvalues $\lambda_1, \dots, \lambda_n$ and the QR-decomposition $A = QR$.

Show:

a) If A is normal and $|\lambda_1| \le |\lambda_2| \le \dots \le |\lambda_n|$, then we have for the diagonal elements of R

$$|\lambda_1| \le |r_{jj}| \le |\lambda_n|, \quad j = 1, \dots, n.$$

b) If $A = Q_1R_1$ is another QR-decomposition of A, then there exists a diagonal matrix $S = \text{diag}(e^{i\varphi_1}, \dots, e^{i\varphi_n})$ with

$$Q_1 = QS, \quad R_1 = S^H R.$$

Problem 35 (no correction)

a) Show that for any matrix $A \in \mathbb{R}^{m\times n}, m \ge n$ a decomposition of the form

$$A = QR$$

can be found with an orthogonal matrix $Q \in \mathbb{R}^{m\times m}$ and a *generalized triangular matrix* $R \in \mathbb{R}^{m\times n}$, i.e. $r_{ij} = 0$ for $1 \le j < i \le n$ and for $i > n$.

b) Use the results in a) to reduce the least square problem

$$\|Ax - b\|_2 \to \min,$$

$A \in \mathbb{R}^{m\times n}, m \ge n$ and $b \in \mathbb{R}^m$ to the system of linear equations

$$\hat{R}x = c$$

with the upper triangular matrix $\hat{R} \in \mathbb{R}^{n\times n}$ and the right side $c \in \mathbb{R}^n$.
Identify $\hat{R}$ and c.

Problem 36 (to be handed in)

a) Transform the matrix

$$A = \begin{pmatrix} 4 & -1 & -1 & 0 \\ -1 & 4 & 0 & -1 \\ -1 & 0 & 4 & -1 \\ 0 & -1 & -1 & 4 \end{pmatrix}$$

with the method of Givens into the tridiagonal matrix A'.
Compute the characteristic polynomial of A' using the recurrence relation and determine the eigenvalues of A.

b) To determine the eigenvalues of the matrix

$$B = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 4 \end{pmatrix}$$

carry out one step of Jacobi's method and give an estimation of the distance between the diagonal elements and the eigenvalues using the theorem of Gerschgorin.

Please hand in your homework problems (No. **34** and **36**) due **Thursday**, **December 21, 2006**, **13:00h**. Put them in the slot marked „Numerische Mathematik I/II/III“ in the Math-Building (20.30), 2nd floor opposite room 112. Please print your name and registration number on your problems.

On **Thursday**, **December 21, 2006**, **14:00-15:30 h** the problems will be discussed in the Neuer Hörsaal (Building 20.40).

Each Thursday a homework assignment will be handed out in the tutorial. The homework assignments are also available for download in the WWW:

http://www.mathematik.uni-karlsruhe.de/ianm3/lehre/numana12006w .